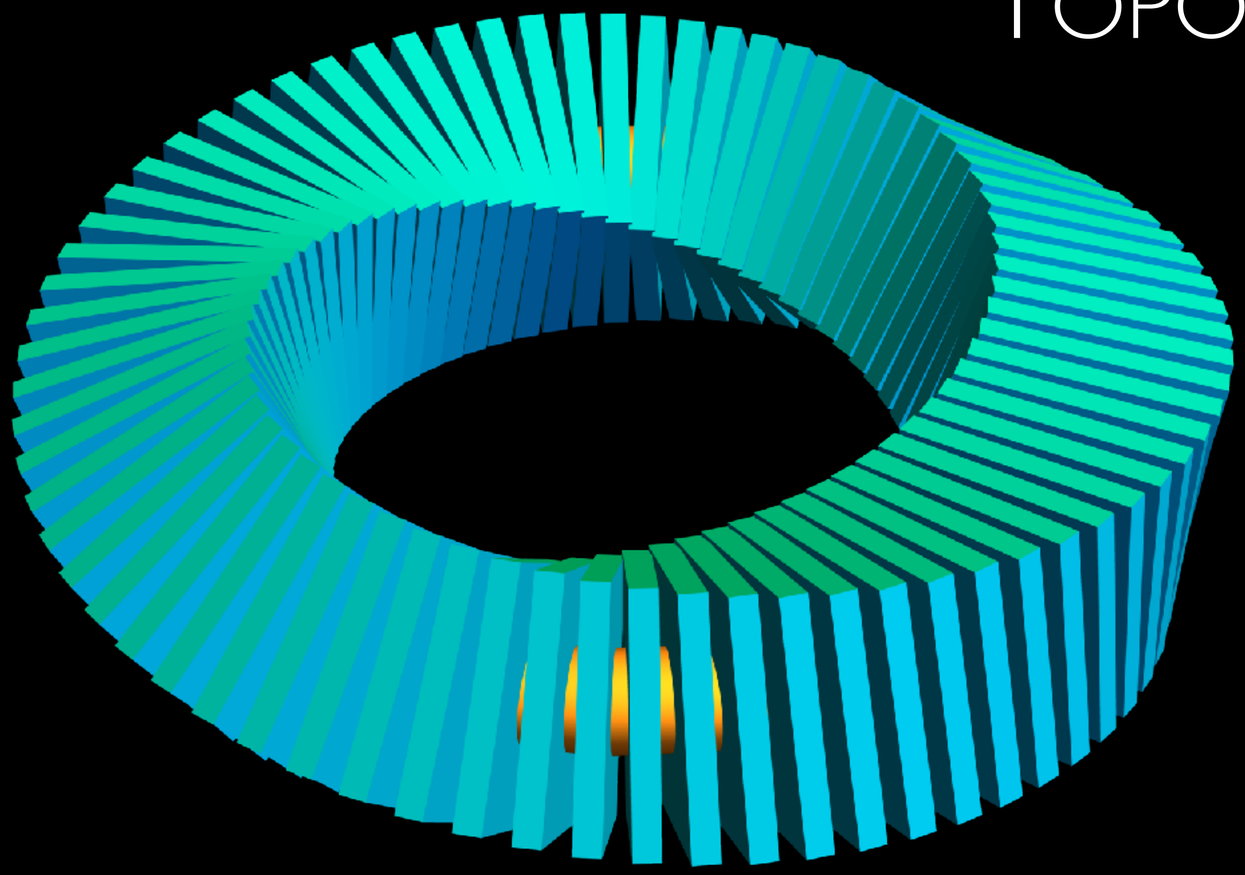
PQChips 2026 (Gdansk, Poland, August 27, 2026)



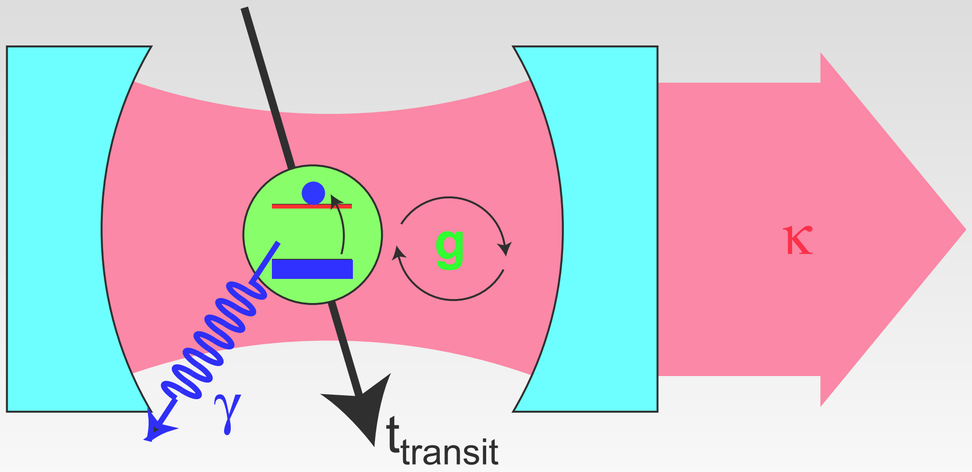
On-Chip Simulation of Topological States of Matter

Mahn-Soo Choi
(Korea University)

$$g(a^\dagger + a)(\sigma^+ + \sigma^-) \rightarrow \underbrace{g_1(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \underbrace{g_2(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}$$

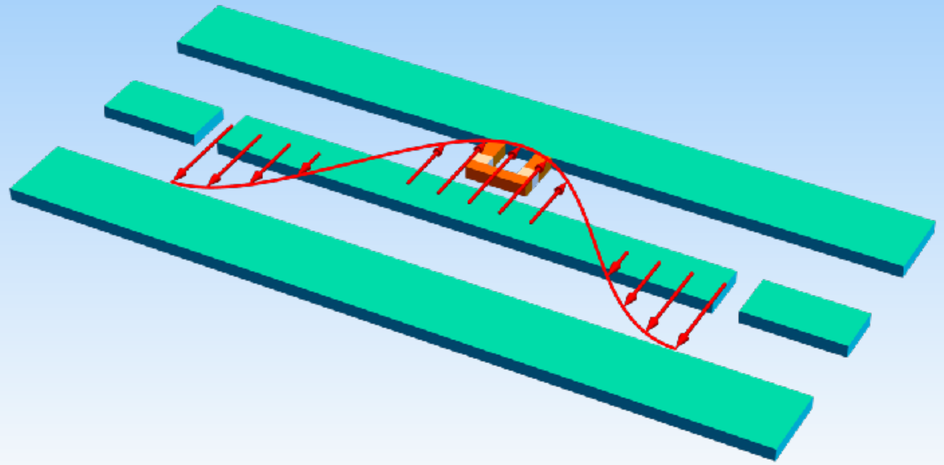
$$\kappa, \gamma \ll g \ll \omega$$

$$g = g_1 = g_2$$

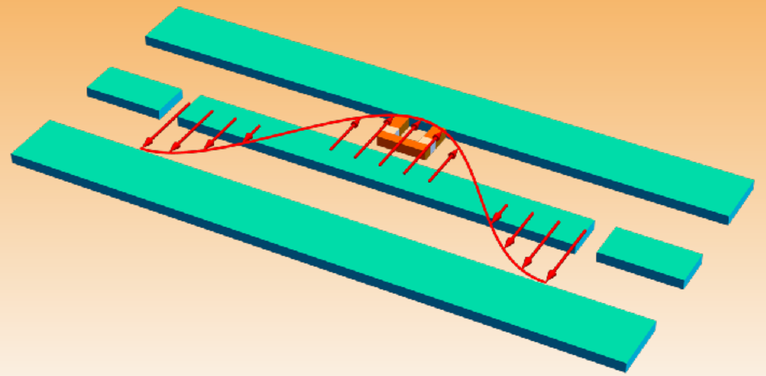


$$g \sim \sqrt{\omega\Omega}$$

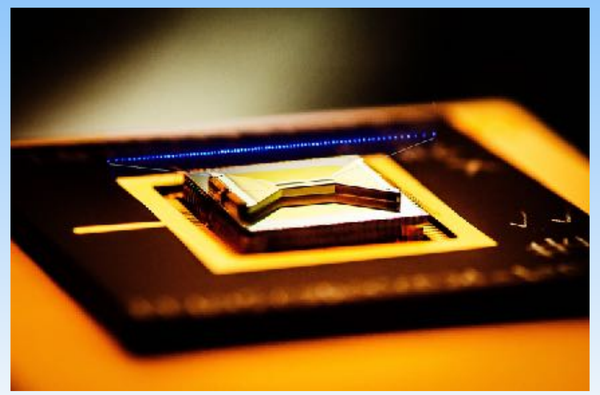
$$g_1 \neq g_2$$



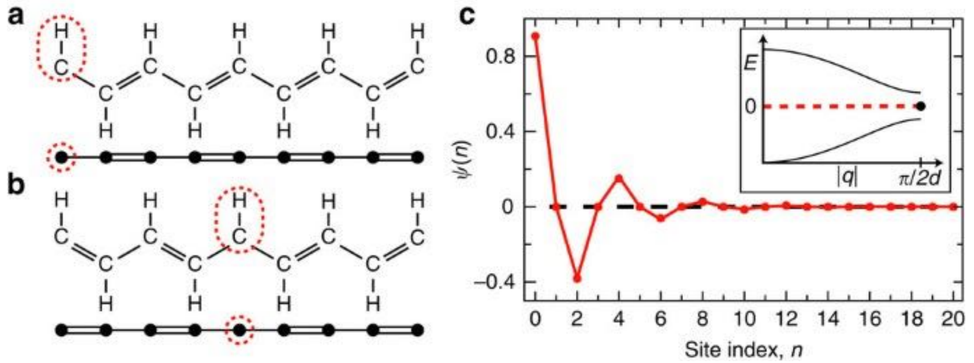
Circuit QED System

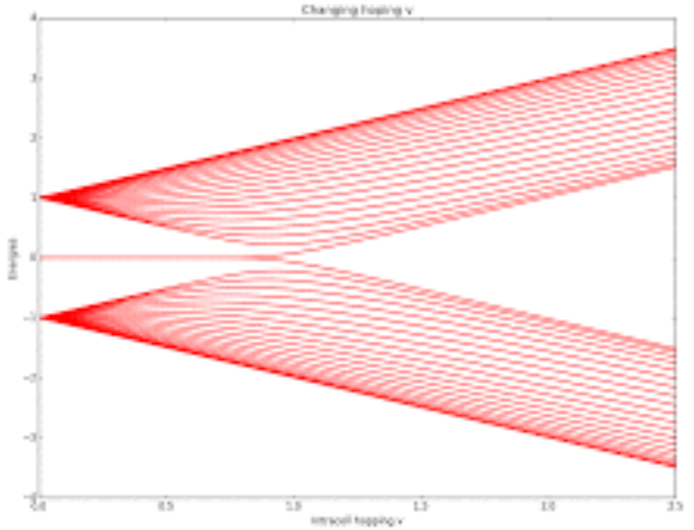


Trapped Ion



Topological Insulator





OUTLINE

- What is a topological state?
  - Chiral symmetry & Topological protection
  - Bulk-boundary correspondence
- Rabi Hamiltonian
- Rabi-Su-Schrieffer-Heeger (RSSH) model
- Topological features in RSSH model

ACKNOWLEDGMENTS

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- Christopher Yale, Brian McFarland, Edward Tortorici, Daniel Lobser, Melissa Revelle, Susan Clark (Sandia Nat'l Lab, USA)

Topological States

Chiral Symmetry & Topological Protection

class					$\delta := d - D$							
s	AZ		T	C	S	0	1	2	3	4	5	6	7
0	A	(unitary)	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	AIII	(chiral unitary)	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	AI	(orthogonal)	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	BDI	(chiral orthogonal)	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
2	D		0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
3	DIII		−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
4	AII	(symplectic)	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
5	CII	(chiral symplectic)	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
6	C		0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
7	CI		+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Dyson (JMP, 1962)
Altland & Zirnbauer (PRB, 1997)
Chiu et al. (RMP, 2017)

Start with Chiral Symmetry

With simple examples

A Super Simple Quiz

Eigenvalues

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

A Super Simple Quiz

Eigenvalues

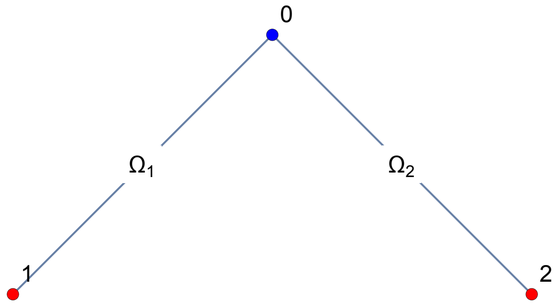
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$E = \pm 1$$

A Simple Quiz

Eigenvalues

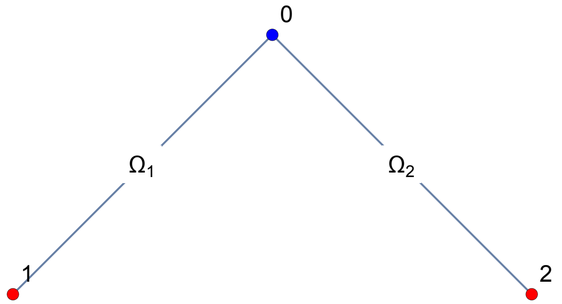
$$\begin{bmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1^* & 0 & 0 \\ \Omega_2^* & 0 & 0 \end{bmatrix}$$



A Simple Quiz

Eigenvalues

$$\begin{bmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1^* & 0 & 0 \\ \Omega_2^* & 0 & 0 \end{bmatrix}$$

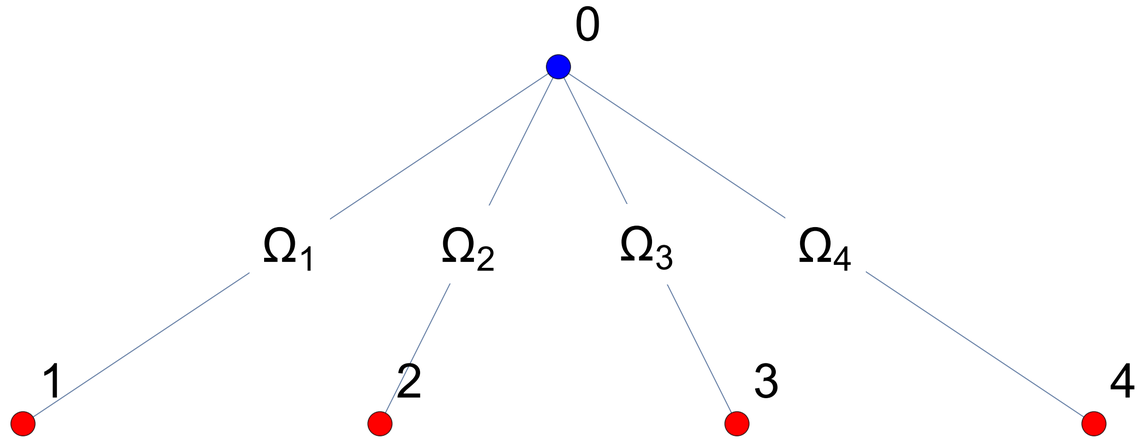


$$E = \pm\sqrt{|\Omega_1|^2 + |\Omega_2|^2}, \quad 0$$

Not-So Simple(?) Quiz

Eigenvalues

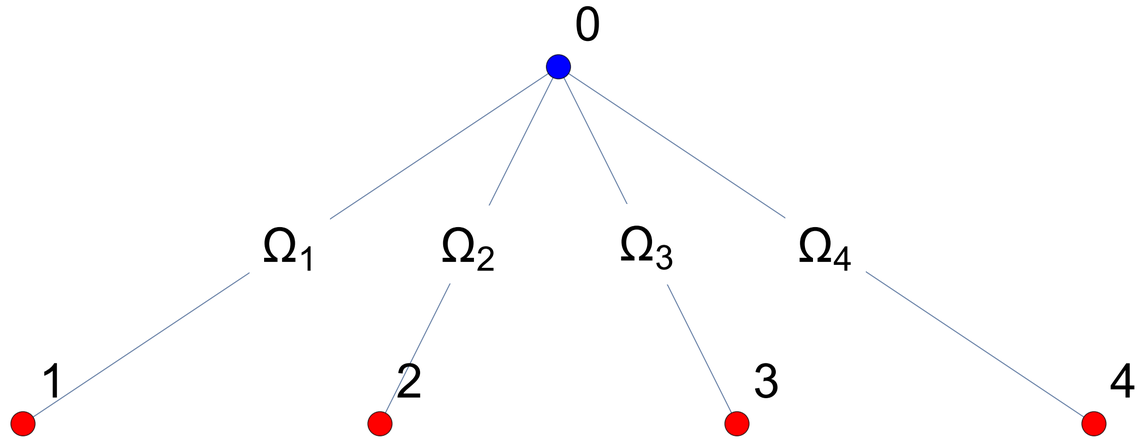
$$\begin{bmatrix} 0 & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 \\ \Omega_1^* & 0 & 0 & 0 & 0 \\ \Omega_2^* & 0 & 0 & 0 & 0 \\ \Omega_3^* & 0 & 0 & 0 & 0 \\ \Omega_4^* & 0 & 0 & 0 & 0 \end{bmatrix}$$



Not-So Simple(?) Quiz

Eigenvalues

$$\begin{bmatrix} 0 & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 \\ \Omega_1^* & 0 & 0 & 0 & 0 \\ \Omega_2^* & 0 & 0 & 0 & 0 \\ \Omega_3^* & 0 & 0 & 0 & 0 \\ \Omega_4^* & 0 & 0 & 0 & 0 \end{bmatrix}$$

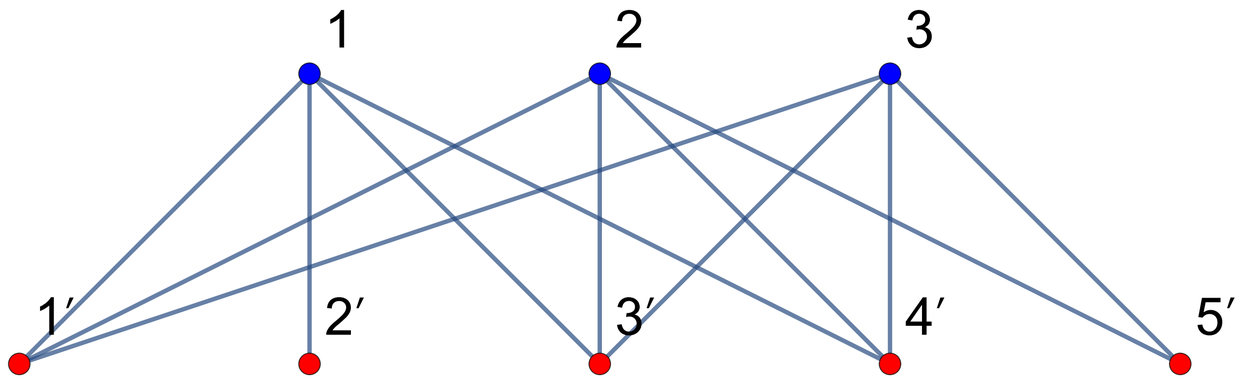


$$E = \pm\sqrt{|\Omega_1|^2 + \cdots + |\Omega_n|^2}, \quad \underbrace{0, \cdots, 0}_{n-1}$$

A Challenging(?) Quiz

How many zero-eigenvalues?

	1	2	3	1′	2′	3′	4′	5′
1	0	0	0	2	1	3	1	0
2	0	0	0	3	0	1	2	3
3	0	0	0	3	0	1	2	1
1′	2	3	3	0	0	0	0	0
2′	1	0	0	0	0	0	0	0
3′	3	1	1	0	0	0	0	0
4′	1	2	2	0	0	0	0	0
5′	0	3	1	0	0	0	0	0



Chiral Symmetry

"Anti-Symmetry"

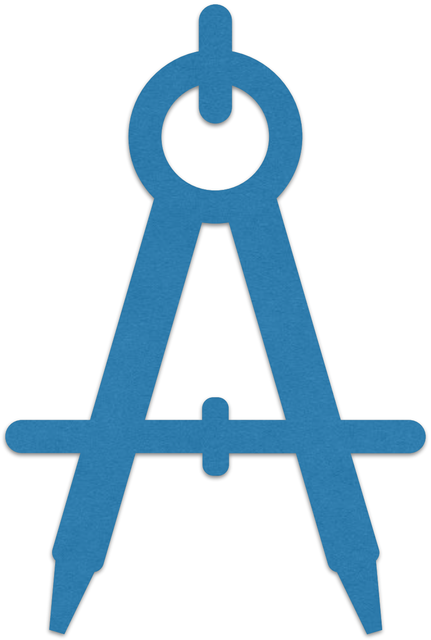
$$\left[\begin{array}{ccc|ccc} & & & s_1 & & \\ & & & & s_2 & \\ & & & & & s_3 \\ & & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline s_1 & & & 0 & 0 & \\ & s_2 & & 0 & 0 & \\ & & s_3 & 0 & 0 & \end{array}\right]$$

Hamiltonian

$$\left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ \hline & & & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{array}\right]$$

Chirality operator

$$\{\hat{H}, \hat{\Xi}\} = 0$$

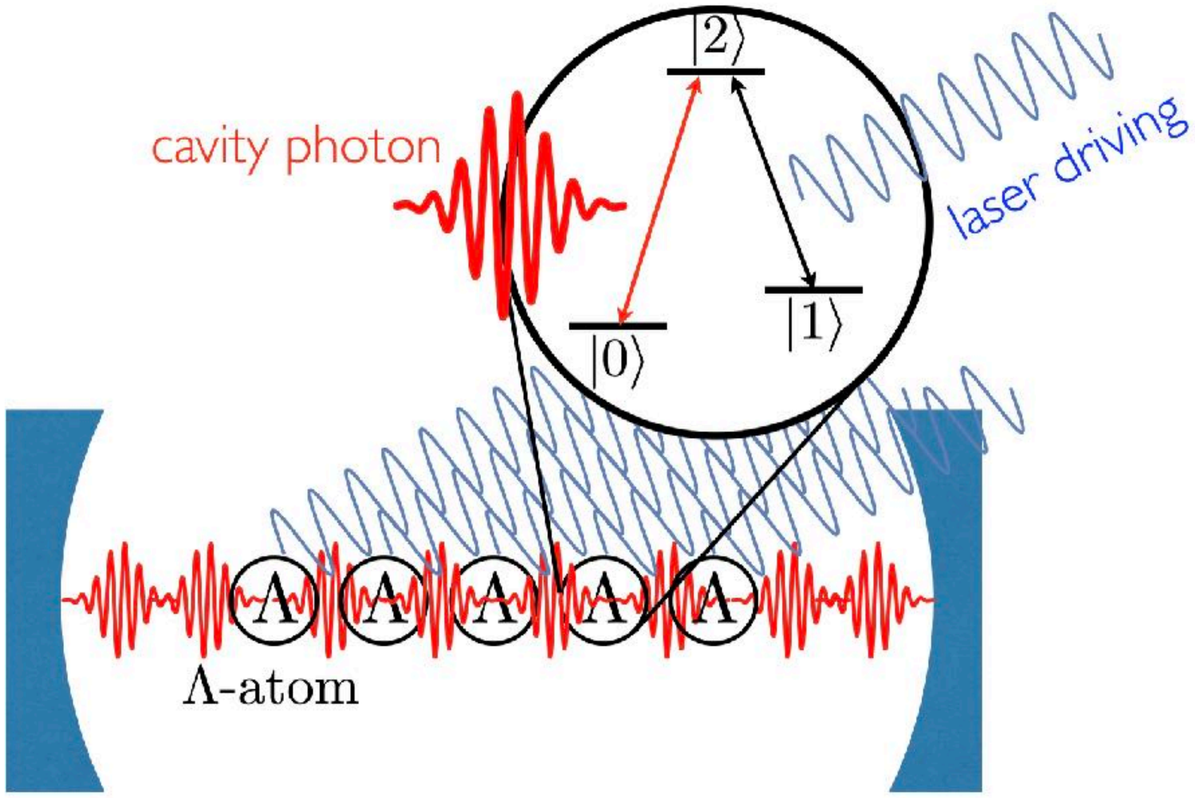


Chiral symmetry
protects zero-energy states!

Physical Example

Chirality-Protected Decoherence-Free Subspace

Chen, Wang, Stefano, Choi (PRA, 2022)



$$
\begin{bmatrix}
0 & 0 & 0 & 0 & \Omega_2 & 1 & 0 & \Omega_1^* & 0 \\
0 & 0 & 0 & 0 & \Omega_1 & 0 & 1 & \Omega_2^* & 0 \\
0 & 0 & 0 & 0 & 0 & \Omega_1 & 0 & 1 & \sqrt{2} \\
0 & 0 & 0 & 0 & 0 & 0 & \Omega_2 & 1 & \sqrt{2} \\
\Omega_2^* & \Omega_1^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \Omega_1^* & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \Omega_2^* & 0 & 0 & 0 & 0 & 0 \\
\Omega_1 & \Omega_2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2} & \sqrt{2} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Where is topology?

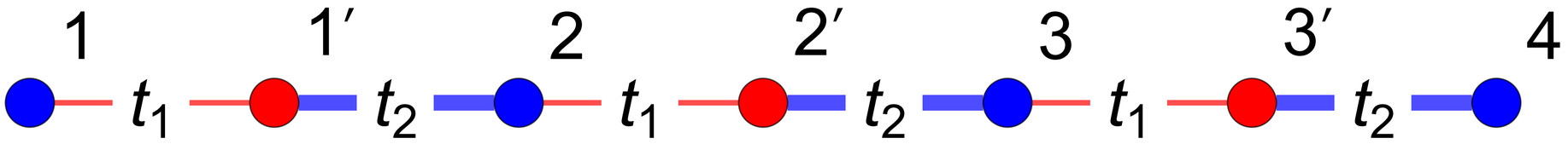
- Topology is about "continuity."
- Therefore, we need to consider a "matter."

Start with a Tight-Binding Model

Su-Schrieffer-Heeger (SSH) Model

SU-SCHRIEFFER-HEEGER (SSH) MODEL

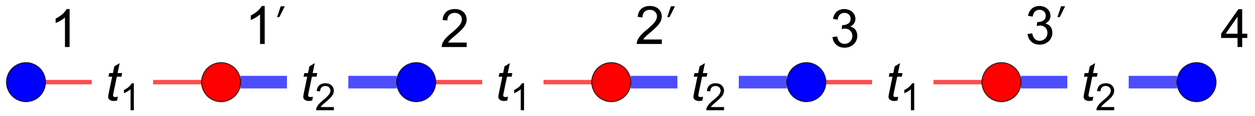
IN FACT, NOT QUITE!



$$\hat{H} = t_1 \sum_{j=1}^{L} a_j^\dagger b_j + t_2 \sum_{j=1}^{L-1} b_j^\dagger a_{j+1} + h.c.$$

Su-Schrieffer-Heeger (SSH) Model

In fact, not quite!

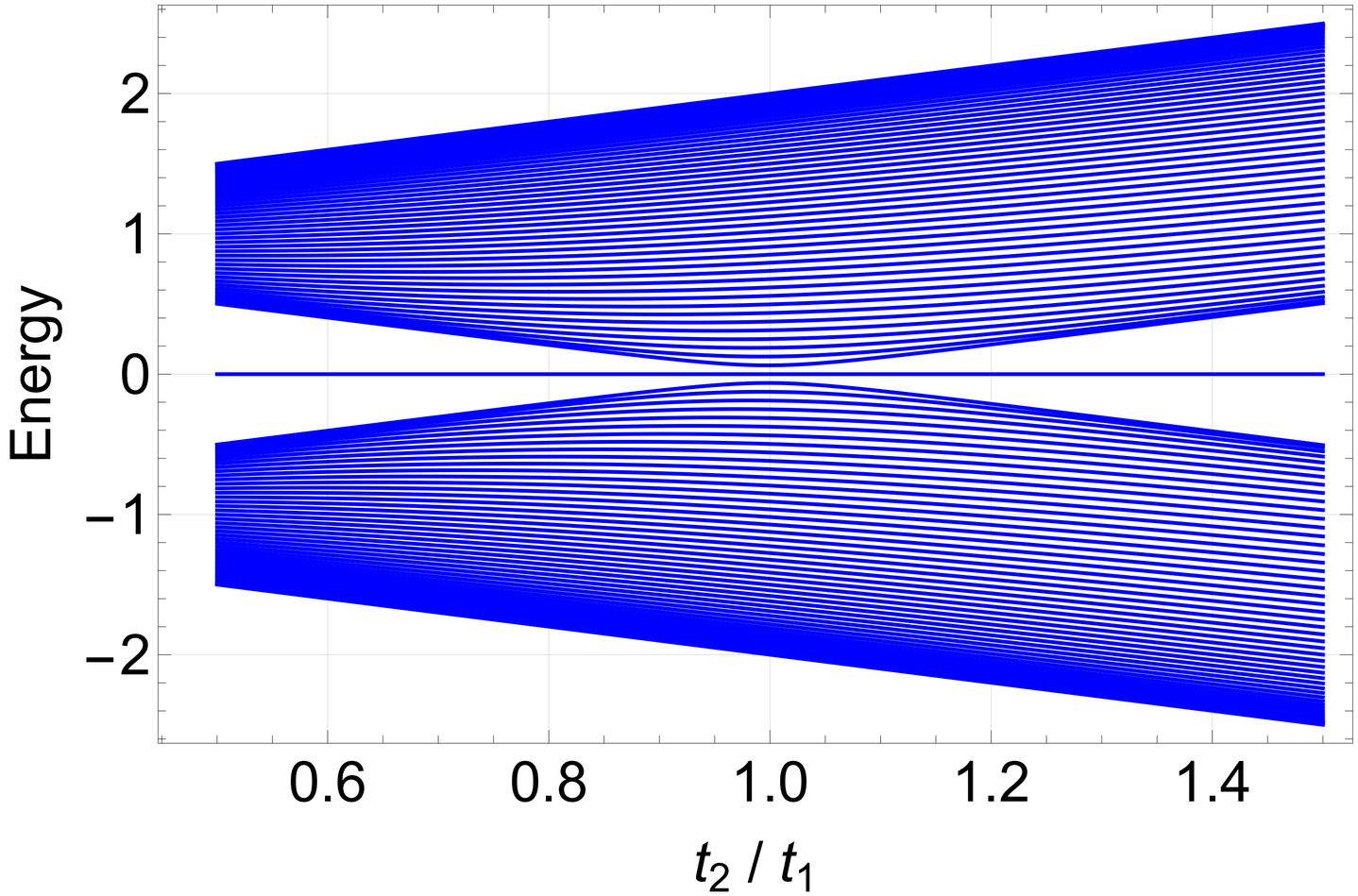


$$
\left[\begin{array}{c||cccc|ccc}
 & a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 \\
\hline\hline
a_1 & 0 & 0 & 0 & 0 & t_1 & 0 & 0 \\
a_2 & 0 & 0 & 0 & 0 & t_2 & t_1 & 0 \\
a_3 & 0 & 0 & 0 & 0 & 0 & t_2 & t_1 \\
a_4 & 0 & 0 & 0 & 0 & 0 & 0 & t_2 \\
\hline\hline
b_1 & t_1 & t_2 & 0 & 0 & 0 & 0 & 0 \\
b_2 & 0 & t_1 & t_2 & 0 & 0 & 0 & 0 \\
b_3 & 0 & 0 & t_1 & t_2 & 0 & 0 & 0
\end{array}\right]
$$

Too Robust, No Transition

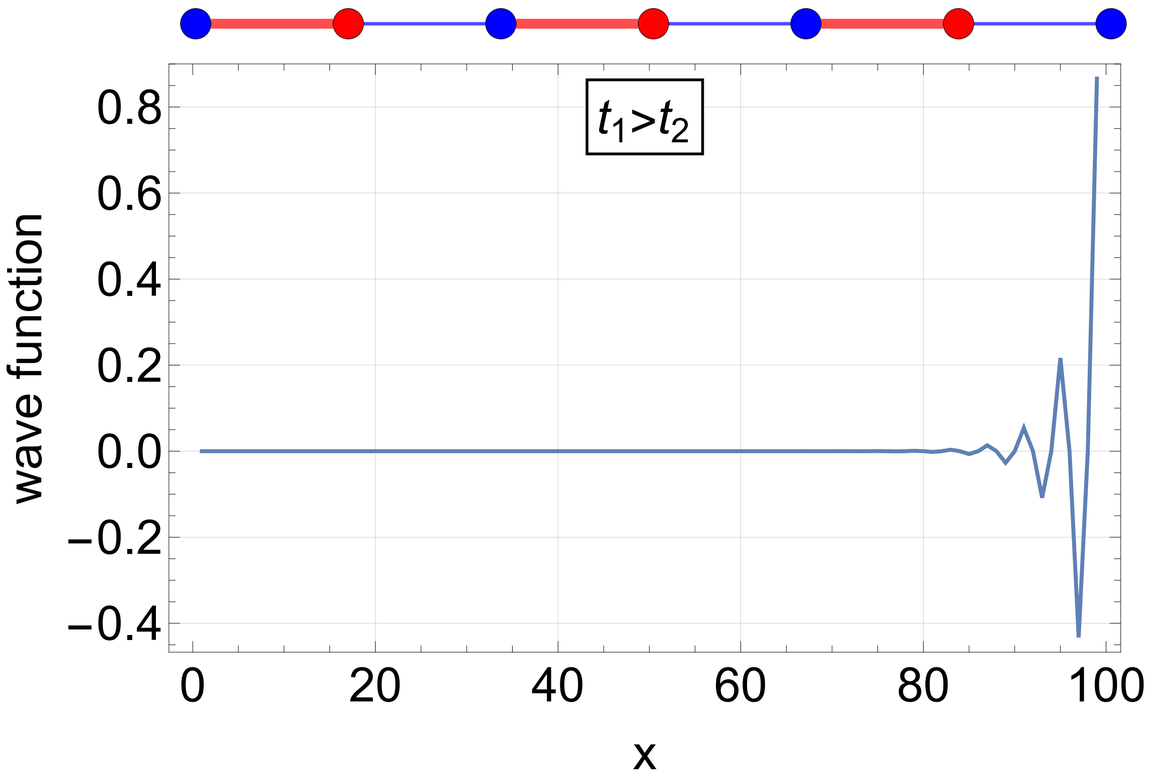
Not-quite-SSH model

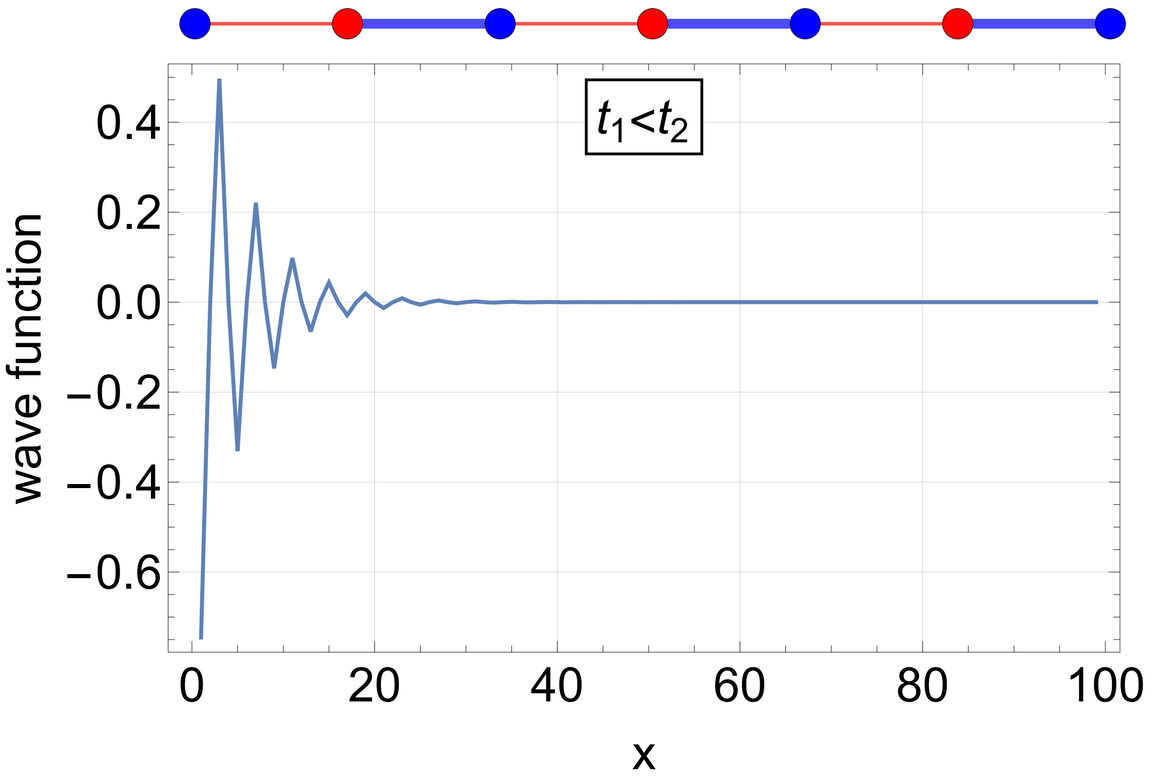
$$
\left[\begin{array}{c||cccc|ccc}
 & a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 \\
\hline\hline
a_1 & 0 & 0 & 0 & 0 & t_1 & 0 & 0 \\
a_2 & 0 & 0 & 0 & 0 & t_2 & t_1 & 0 \\
a_3 & 0 & 0 & 0 & 0 & 0 & t_2 & t_1 \\
a_4 & 0 & 0 & 0 & 0 & 0 & 0 & t_2 \\
\hline
b_1 & t_1 & t_2 & 0 & 0 & 0 & 0 & 0 \\
b_2 & 0 & t_1 & t_2 & 0 & 0 & 0 & 0 \\
b_3 & 0 & 0 & t_1 & t_2 & 0 & 0 & 0
\end{array}\right]
$$



THERE IS SOMETHING IN THE ZERO-ENERGY STATE!

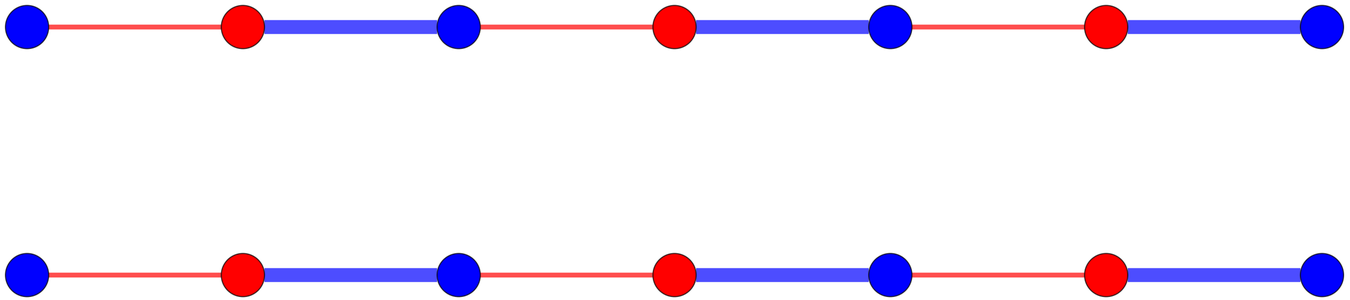
NOT-QUITE-SSH MODEL





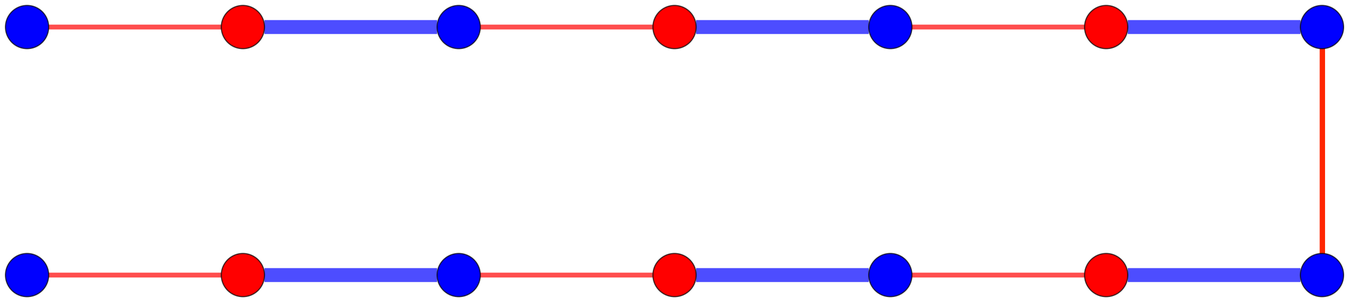
A "True" SSH Model

Combining two subchains



A "True" SSH Model

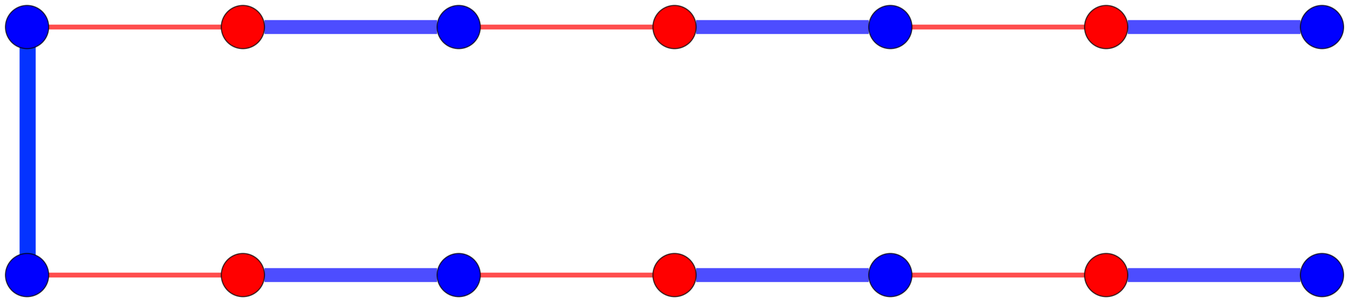
Combining two subchains



Two edge states at zero-energy

A "True" SSH Model

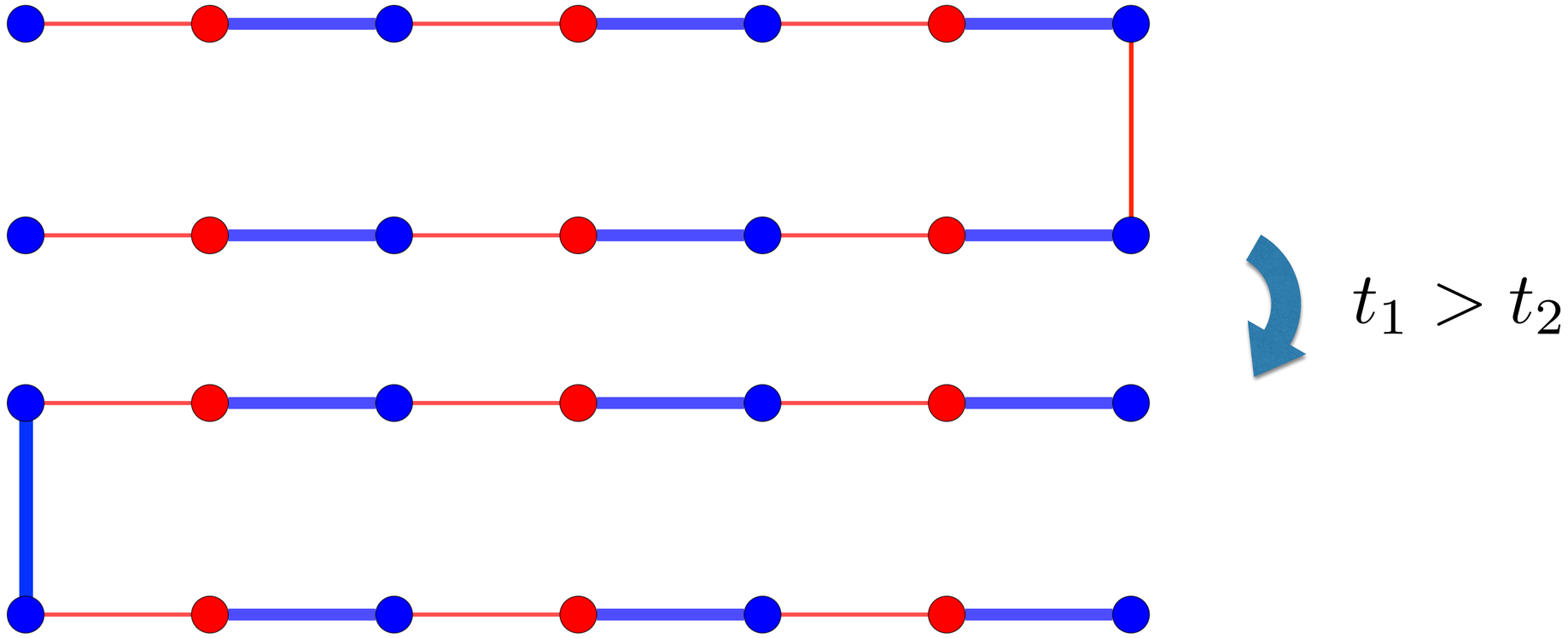
Combining two subchains



No edge state, no zero-energy state

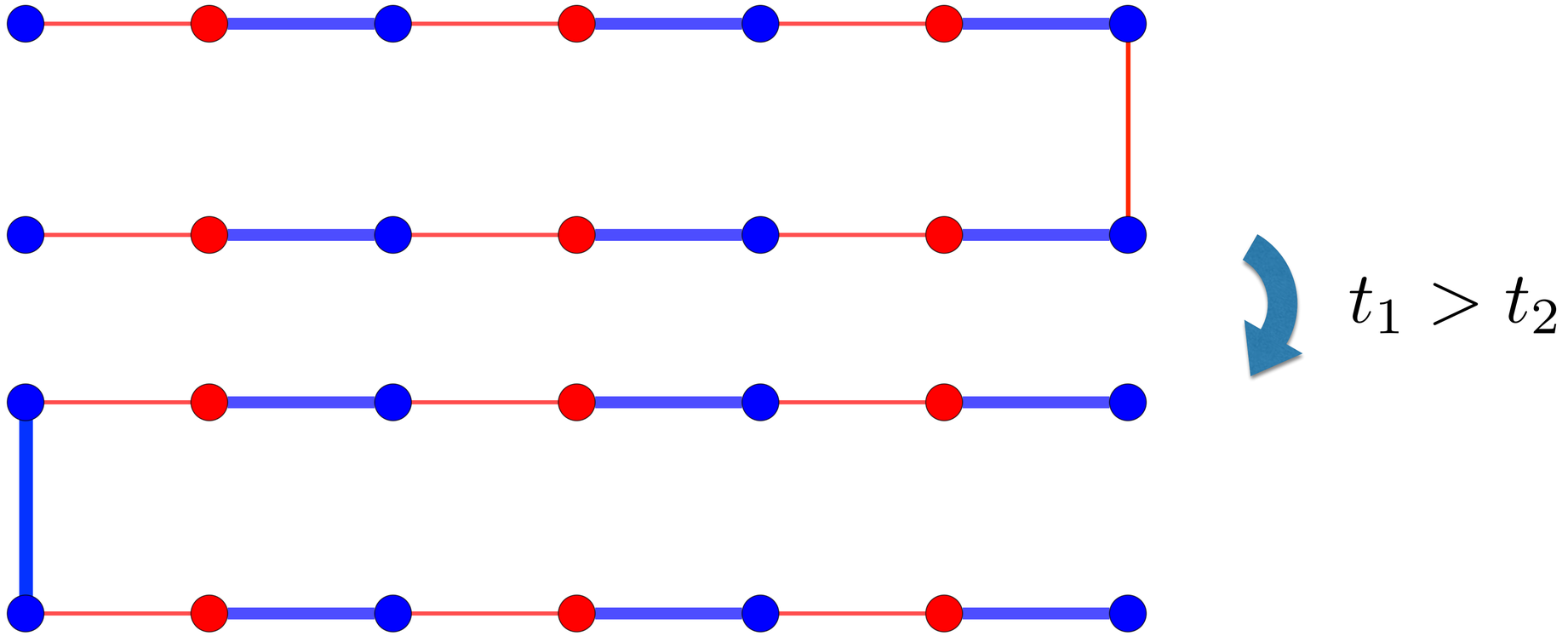
A "True" SSH Model

Just changing the ratio in a full chain



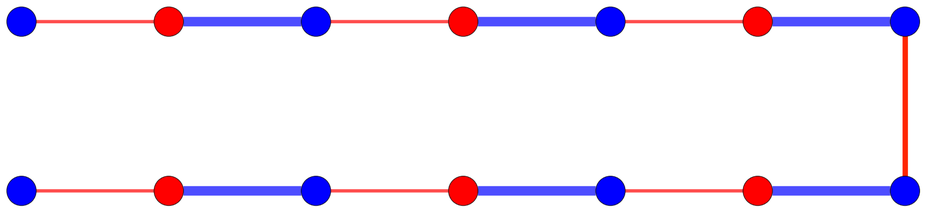
Some Transition!

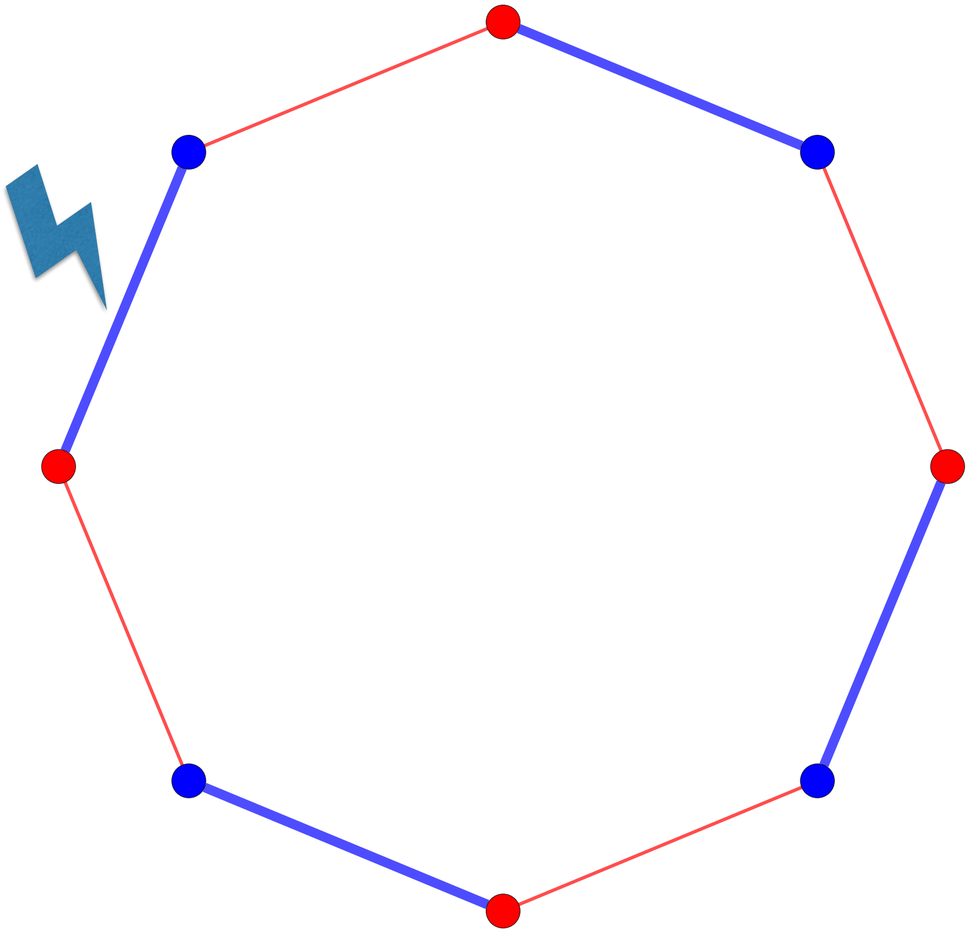
Where does it come from?



The Periodic SSH Model

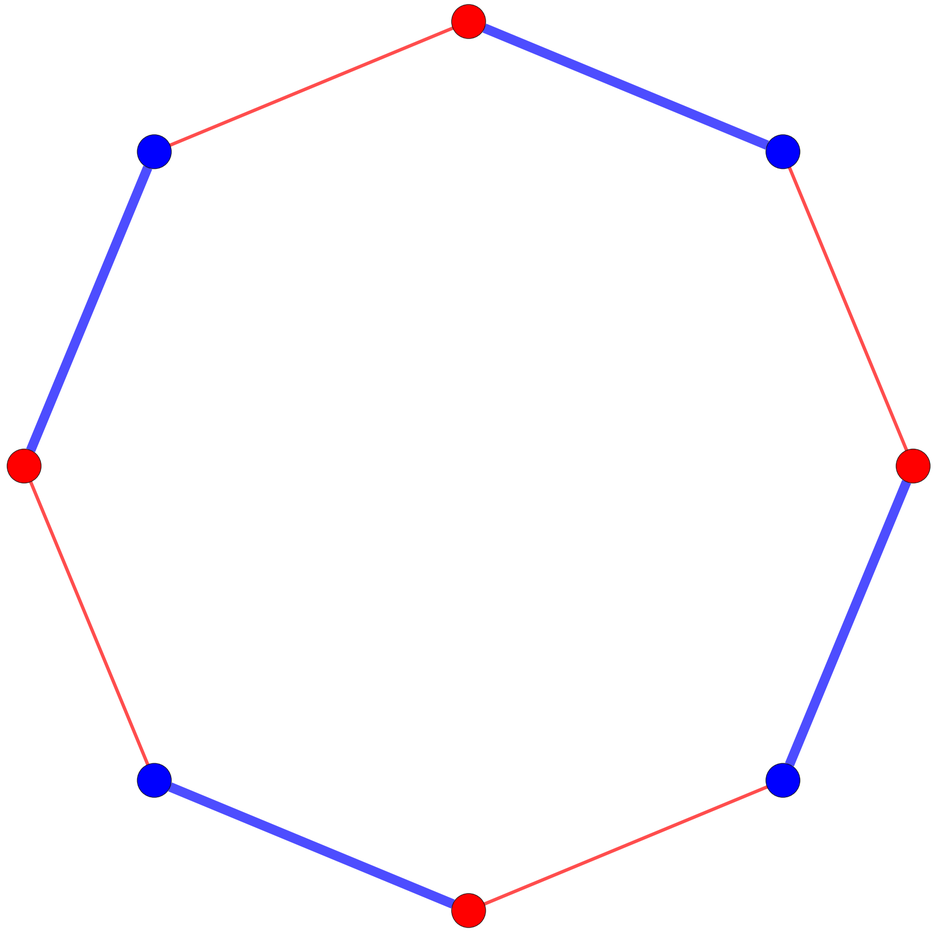
Before it is broken to an open chain





The Periodic SSH Model

Before it is broken to an open chain



$$\hat{H} = t_1 \sum_{j=1}^{L} a_j^\dagger b_j + t_2 \sum_{j=1}^{L-1} b_j^\dagger a_{j+1} + h.c.$$

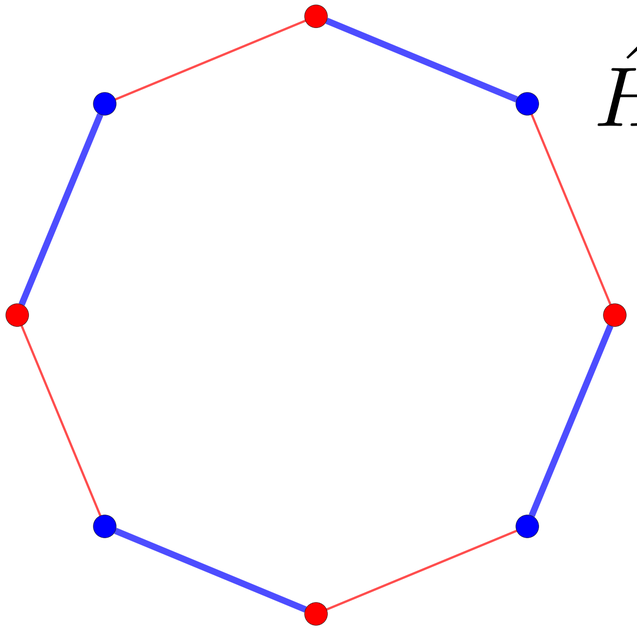
$$\hat{a}_j = \frac{1}{\sqrt{L}} \sum_k e^{ikx_j} \psi_A(k)$$

$$\hat{b}_j = \frac{1}{\sqrt{L}} \sum_k e^{ikx_j} \psi_B(k)$$

The Periodic SSH Model

Before it is broken to an open chain

$$\hat{H} = t_1 \sum_{j=1}^{L} a_j^\dagger b_j + t_2 \sum_{j=1}^{L-1} b_j^\dagger a_{j+1} + h.c.$$

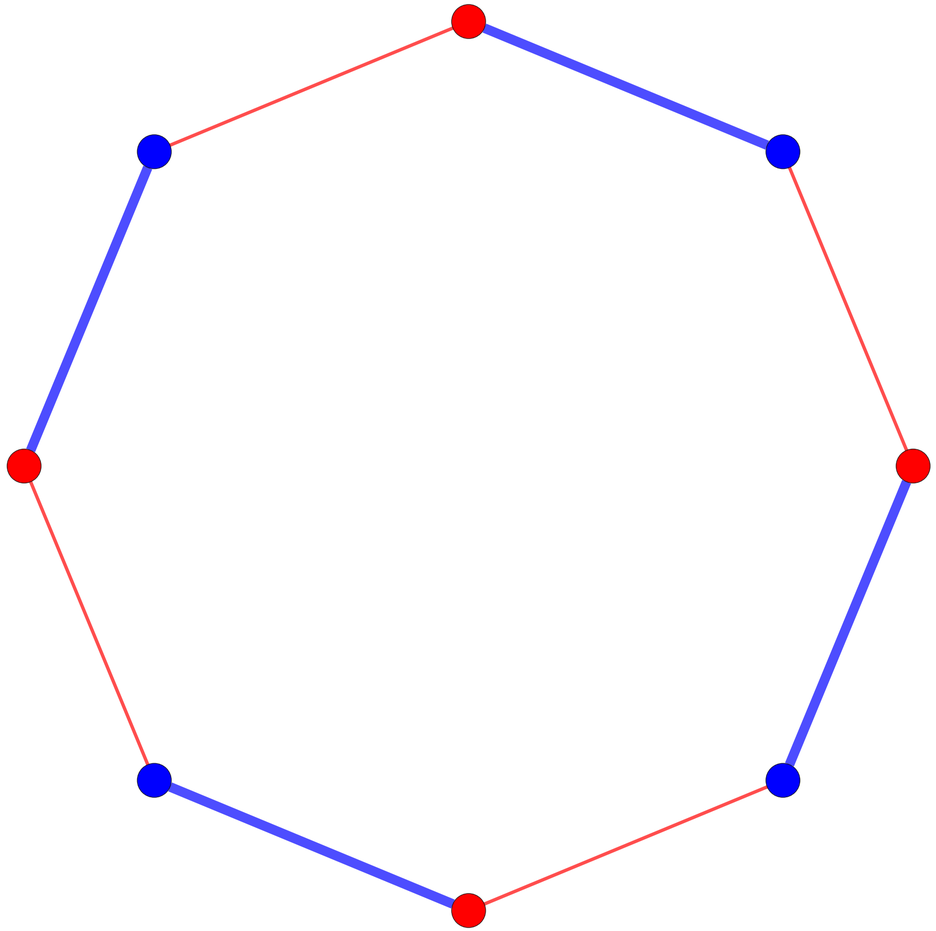


$$\hat{H} = \frac{1}{L} \sum_k \begin{bmatrix} \psi_A^\dagger(k) & \psi_B^\dagger(k) \end{bmatrix} \begin{bmatrix} 0 & T(k) \\ T^*(k) & 0 \end{bmatrix} \begin{bmatrix} \psi_A(k) \\ \psi_B(k) \end{bmatrix}$$

$$T(k) = t_1 - t_2 e^{-ik}$$

The Periodic SSH Model

Before it is broken to an open chain



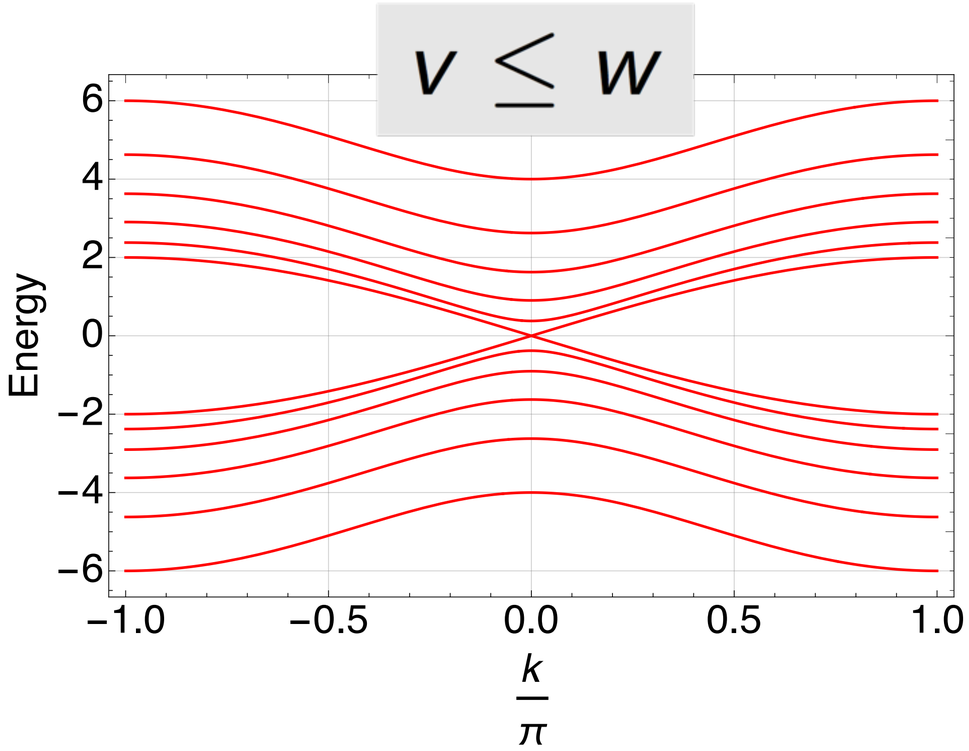
$$H(k) := \begin{bmatrix} 0 & T(k) \\ T^*(k) & 0 \end{bmatrix}$$

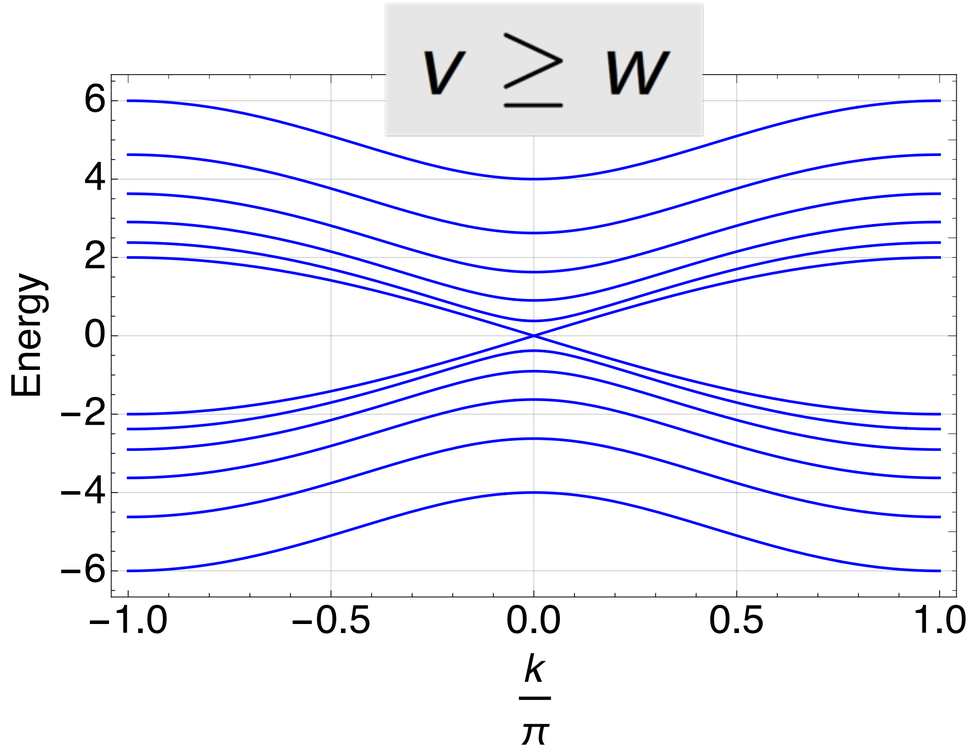
$$H(k) = \boldsymbol{B}(k) \cdot \boldsymbol{S}\,,$$

$$\boldsymbol{B}(k) = (t_1 - t_2 \cos k, -t_2 \sin k, 0)$$

SU-SCHRIEFFER-HEEGER MODEL

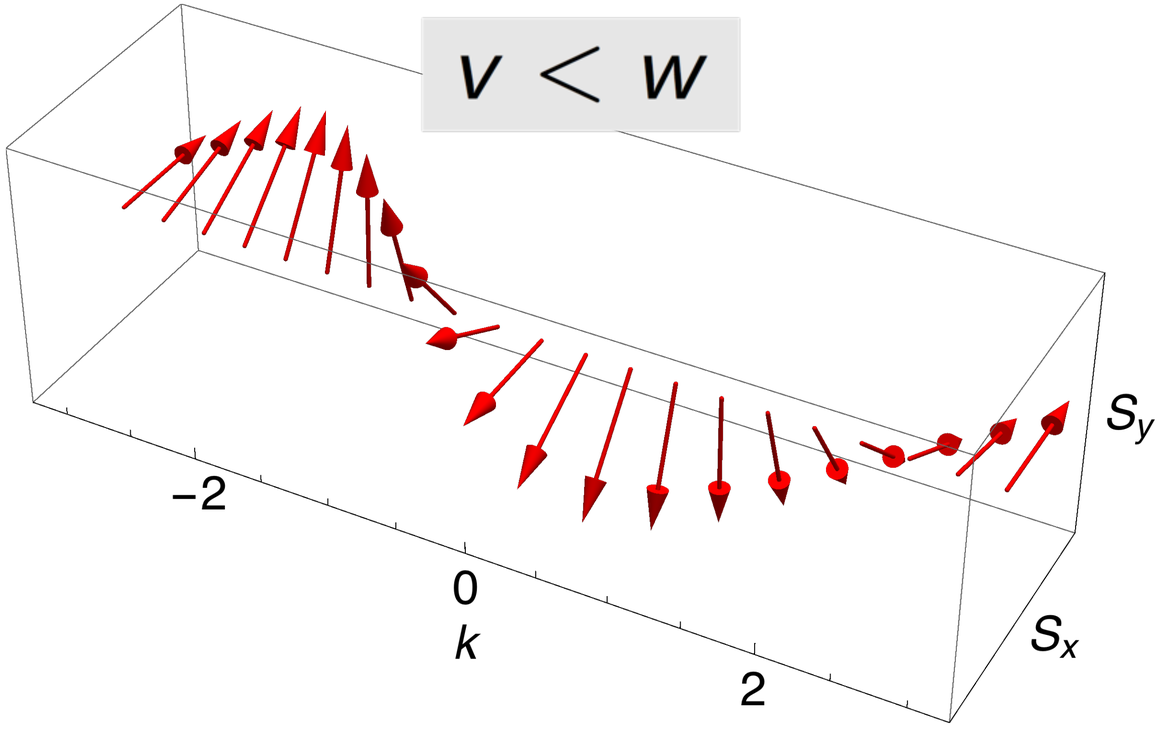
(1D SPINLESS FERMION ON A BIPARTITE LATTICE)

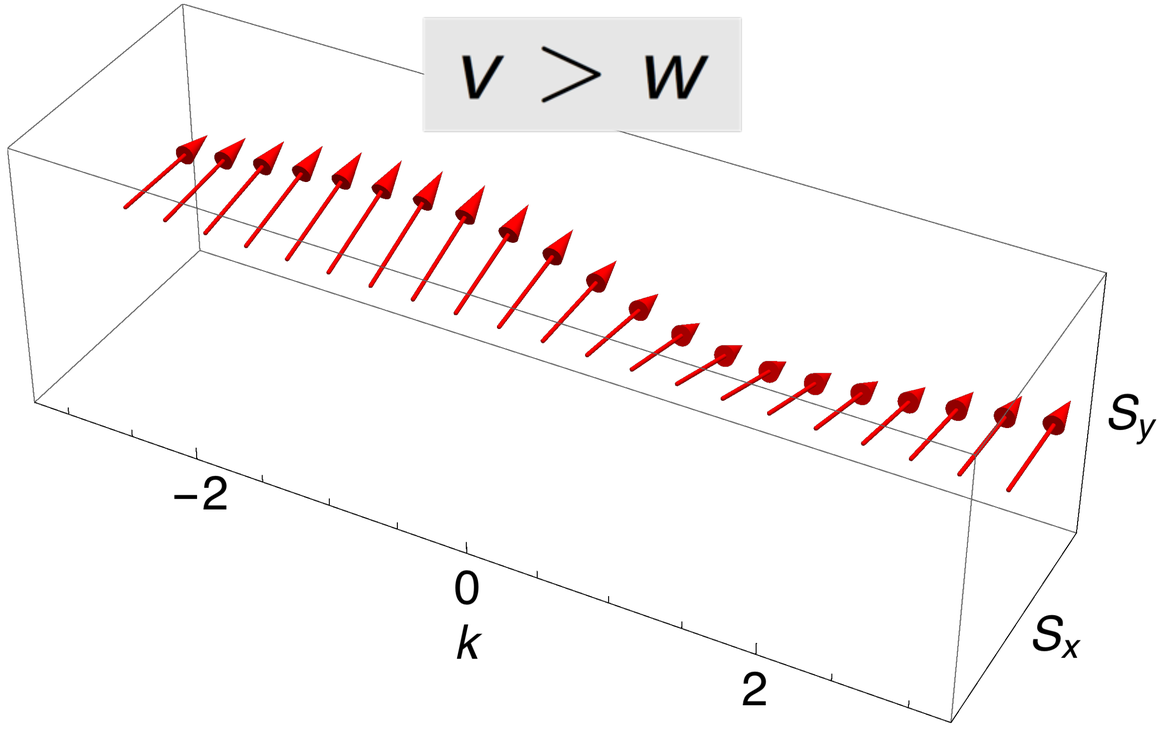




SU-SCHRIEFFER-HEEGER MODEL

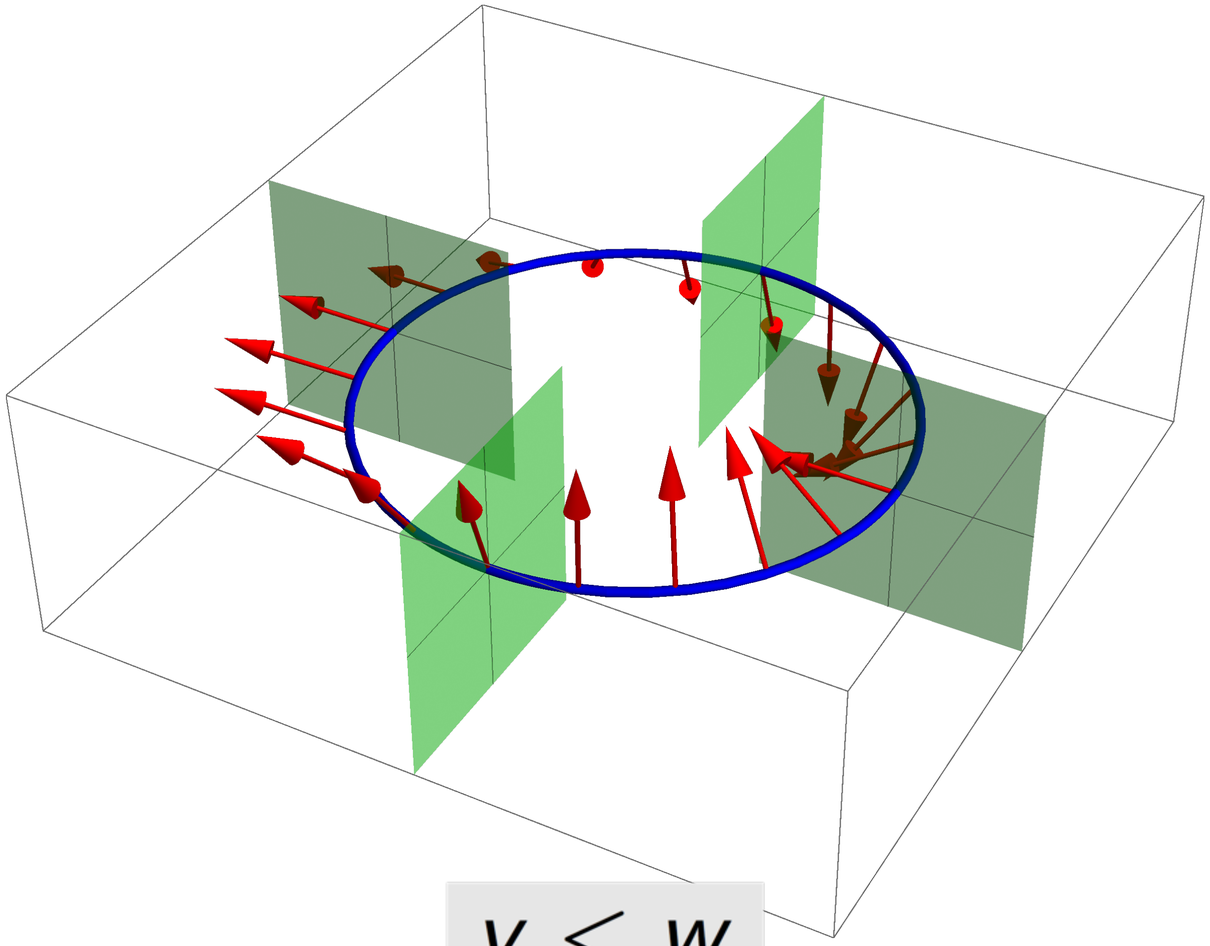
(1D SPINLESS FERMION ON A BIPARTITE LATTICE)



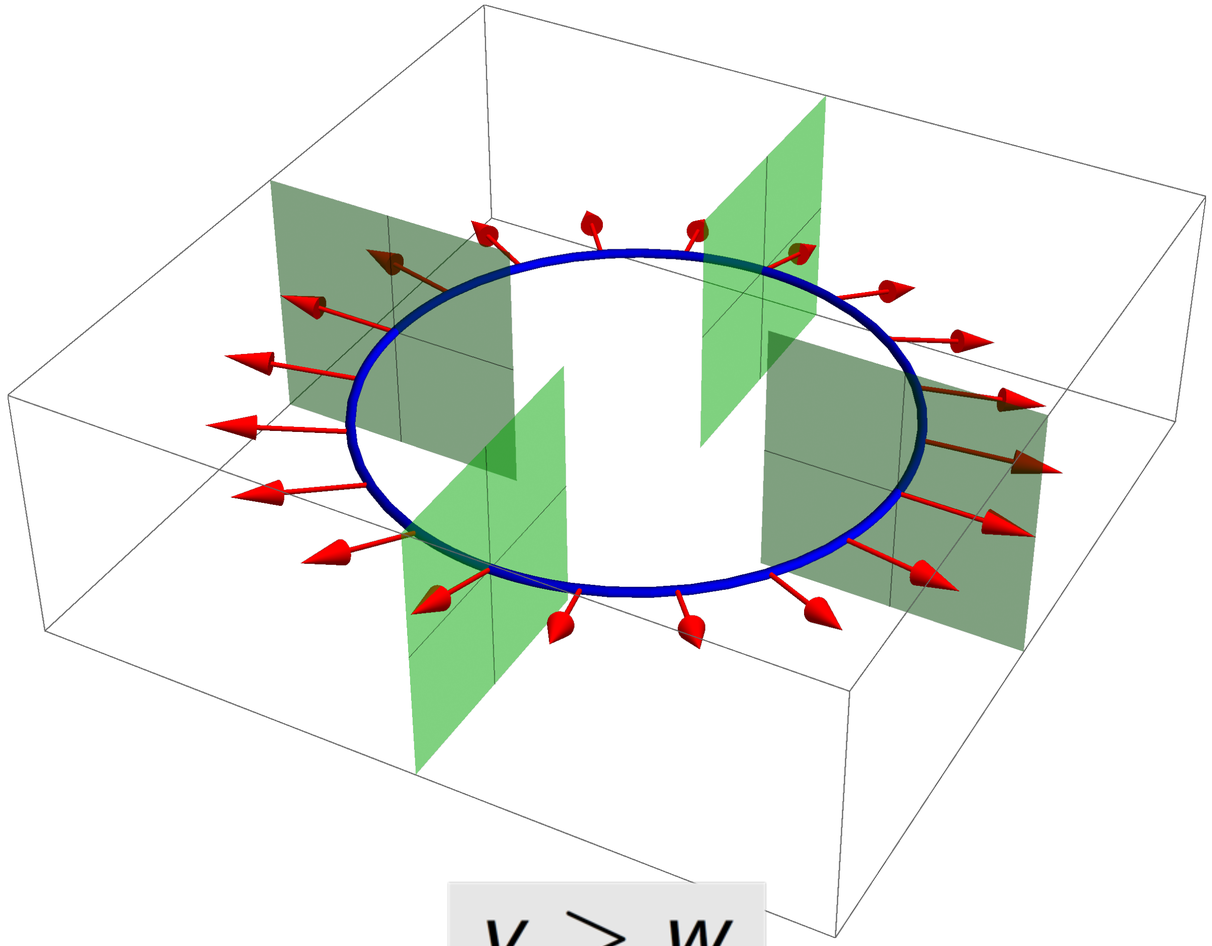


Su-Schrieffer-Heeger Model

(1D Spinless Fermion on a Bipartite Lattice)



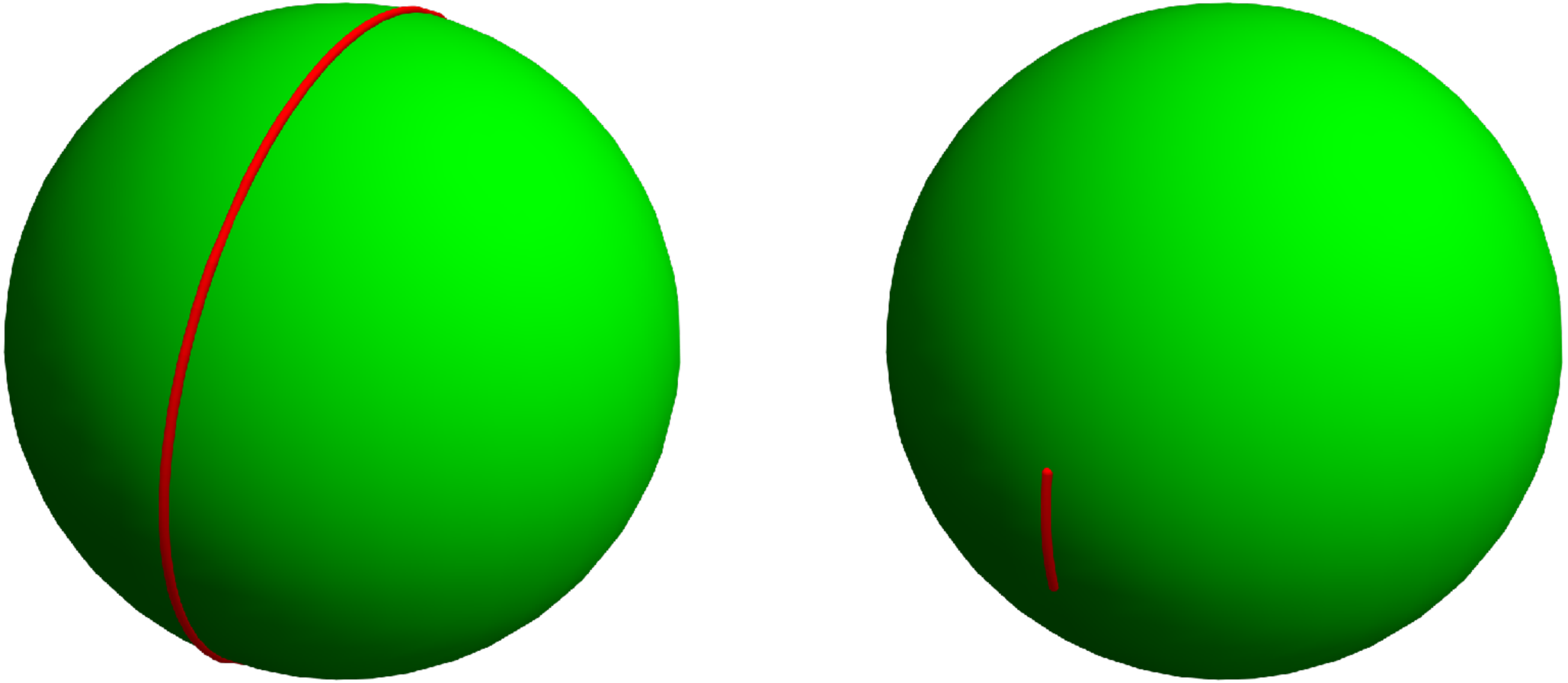
$v < w$



$v > w$

Su-Schrieffer-Heeger Model

(1D Spinless Fermion on a Bipartite Lattice)



Chern Number

(topological quantum number)

$$\hat{H} = \sum_{k \in \mathrm{BZ}} \hat{\psi}^\dagger(k) H(k) \hat{\psi}(k)\,, \quad H(k) := \begin{bmatrix} 0 & T(k) \\ T^*(k) & 0 \end{bmatrix}, \quad \hat{\psi}(k) := \begin{bmatrix} \psi_A(k) \\ \psi_B(k) \end{bmatrix}$$

$$H(k) = \boldsymbol{T} \cdot \boldsymbol{\sigma}\,, \quad \boldsymbol{T}(k) := \begin{bmatrix} v - w\cos k \\ -w \sin k \\ 0 \end{bmatrix}$$

$$C = \int \frac{d^2\boldsymbol{k}}{4\pi} \left[\hat{\boldsymbol{T}} \cdot (\partial_{k_x} \hat{\boldsymbol{T}} \times \partial_{k_y} \hat{\boldsymbol{T}}) \right]$$

Bulk-Boundary Correspondence

Topologically non-trivial state in a bulk =
Zero-energy states at the boundary

Summary So Far

Chiral Symmetry & Topological States

- Chiral symmetry protects zero-energy states.
- Chiral symmetry leads to a *topological classification* of states.
  - The topological states are revealed most clearly by the *bulk-boundary correspondence*.

- Other anti-unitary symmetries (time-reversal and particle-hole) also lead to topological classifications of states.

Asymmetric Rabi Hamiltonian

Chiral Symmetry Kicks In

Rotating vs Counter-Rotating Terms

(Asymmetric Rabi Hamiltonian)

$$g(\hat{a}^\dagger + \hat{a})(\hat{\sigma}^+ + \hat{\sigma}^-) = g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+) + g(\hat{a}^\dagger\hat{\sigma}^+ + \hat{a}\hat{\sigma}^-)$$

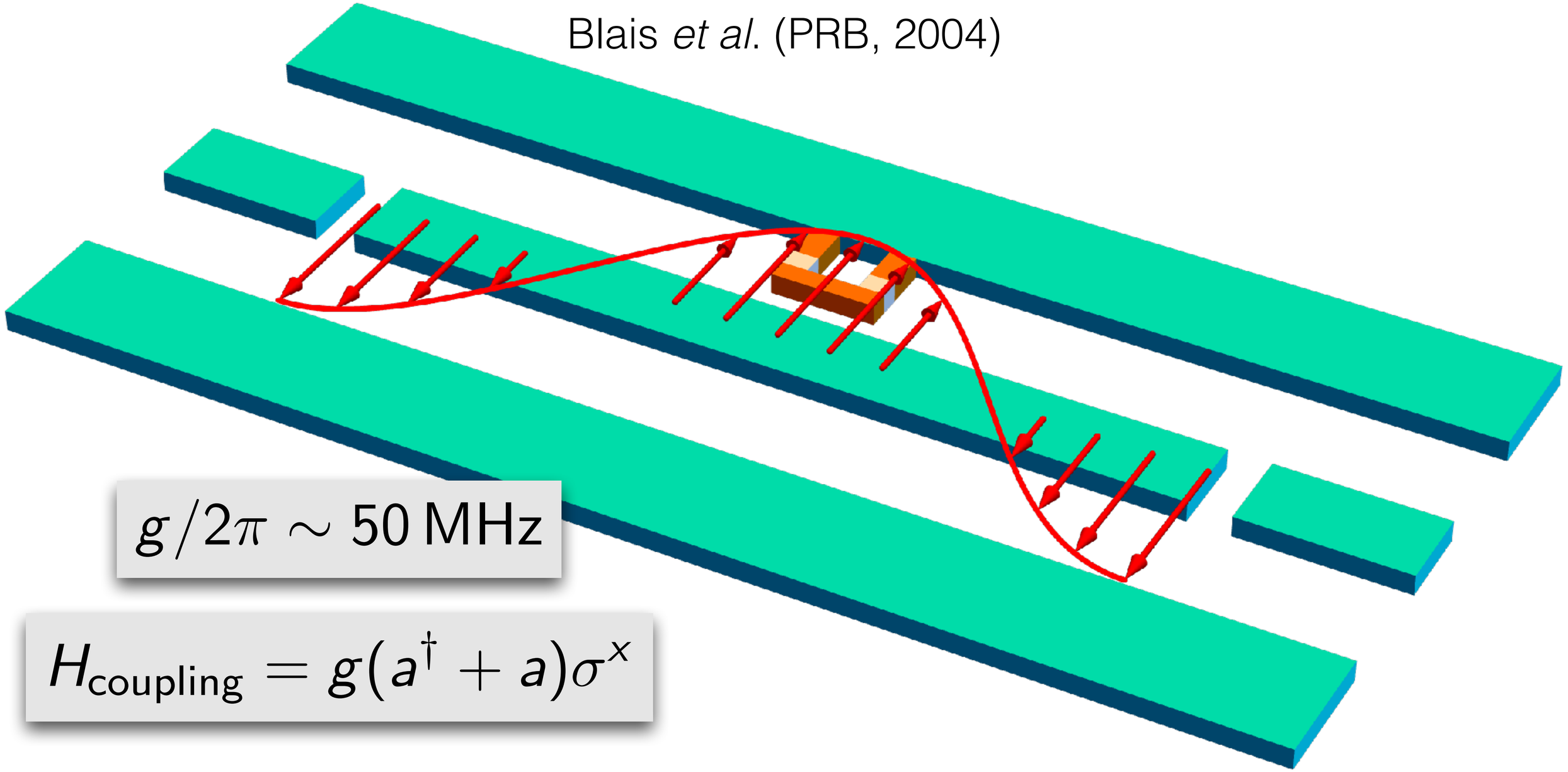
$$\rightarrow g_1 \underbrace{(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)}_{\text{rotating}} + g_2 \underbrace{(\hat{a}^\dagger\hat{\sigma}^+ + \hat{a}\hat{\sigma}^-)}_{\text{counter-rotating}}$$

Erlingsson et al. (PRB, 2010), Schliemann et al. (PRB, 2003)
Tomka et al. (PRA, 2014), Schen et al. (APB, 2014), …

Physical Implementations

Circuit QED System

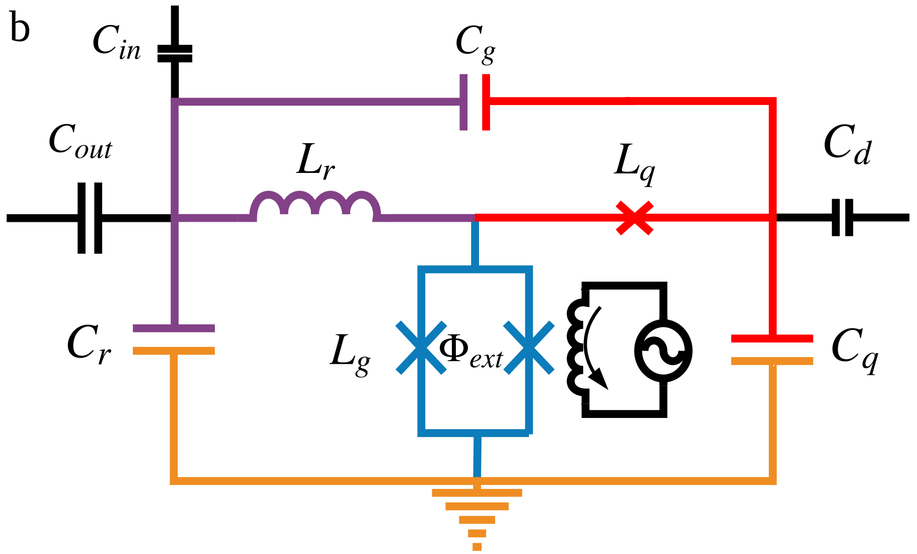
Blais *et al*. (PRB, 2004)



$g/2\pi \sim 50\,\mathrm{MHz}$

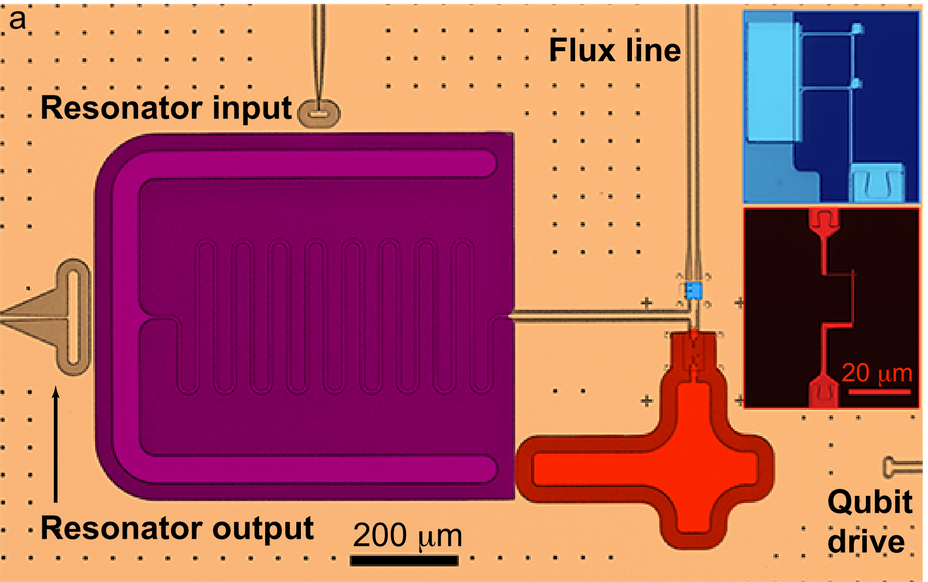
$H_{\mathrm{coupling}} = g(a^\dagger + a)\sigma^x$

Circuit QED Implementation



$$H = \omega a^\dagger a + \frac{\Omega}{2}\sigma_z + g_r(a^\dagger\sigma^- + a\sigma^+) + g_b(a^\dagger\sigma^+ + a\sigma^-)$$

$$g_{r,b} := \frac{L_g(\Phi_{\text{ext}})}{2}\sqrt{\frac{\omega\Omega}{L_r L_q}} \mp \frac{C_g}{2}\sqrt{\frac{\omega\Omega}{C_r C_q}}$$

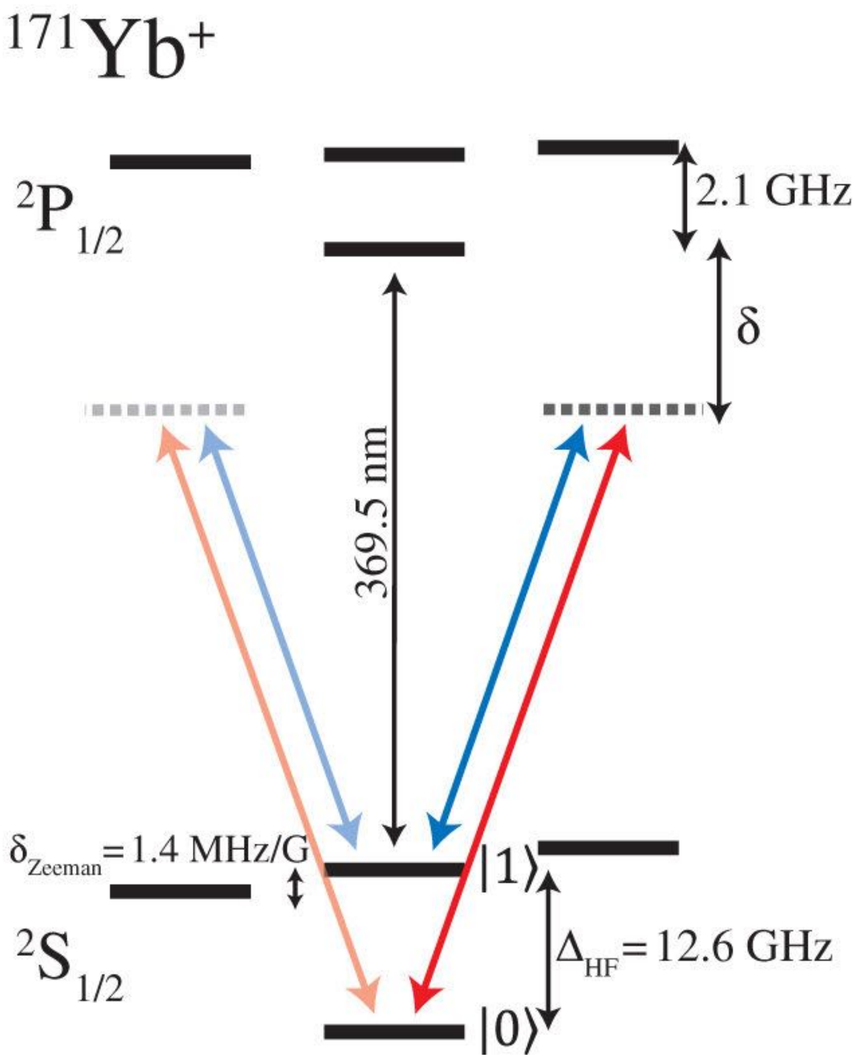


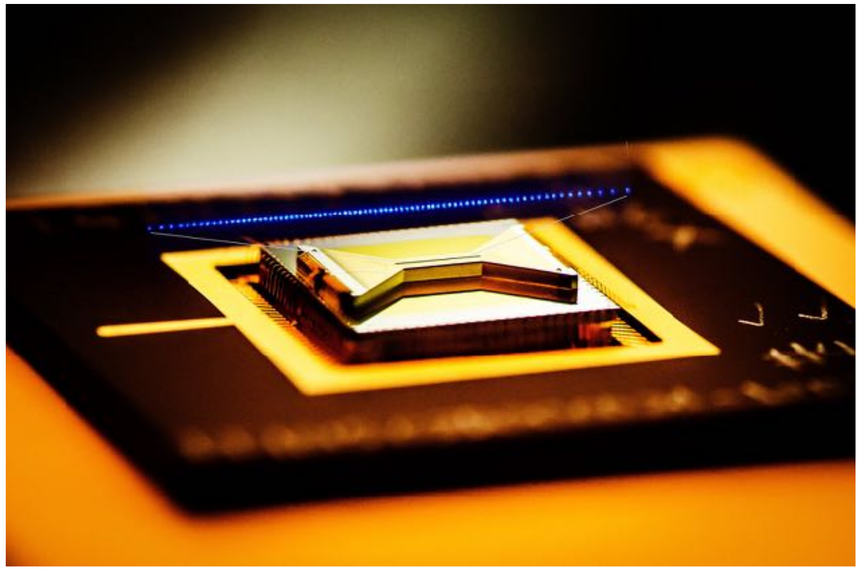
$$L_g(\Phi_{\text{ext}}) := \frac{L_{g0}}{|\cos(\pi\Phi_{\text{ext}}/\Phi_0)|}$$

Lu et al. (PRL, 2017)

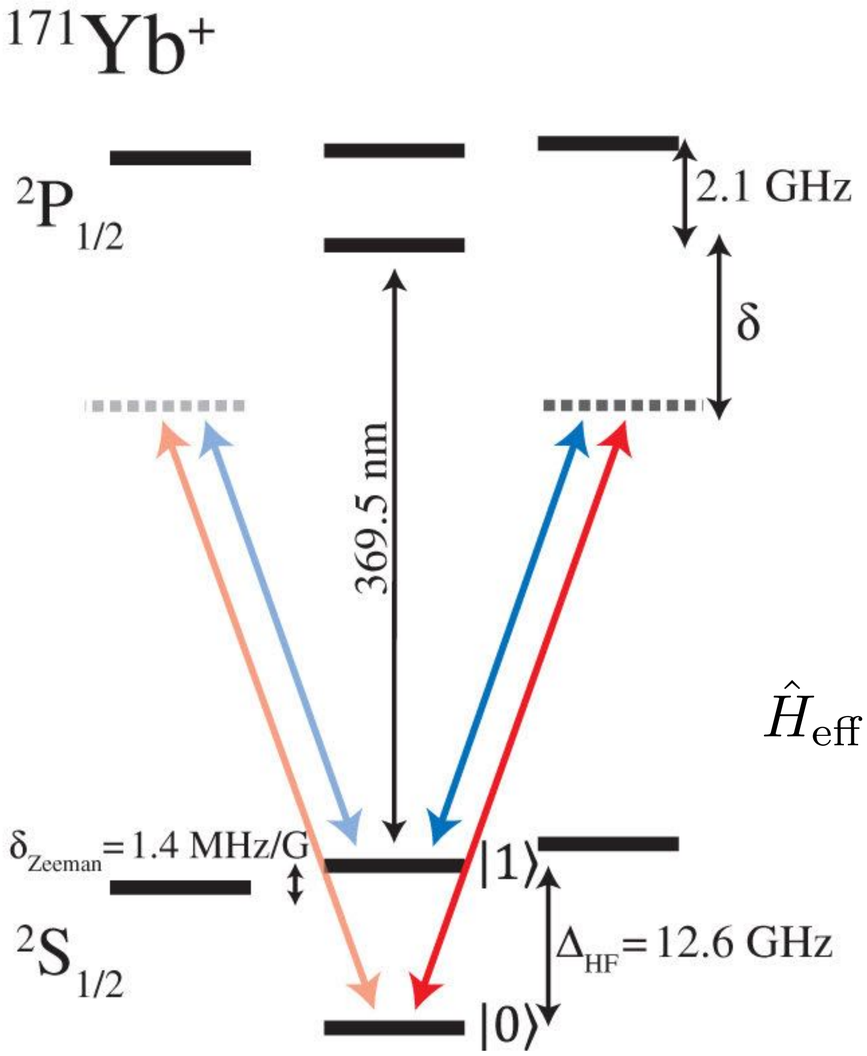
Chen et al. (PRL, 2014)
McKay et al. (PRAppl., 2016)

Trapped Ion





$$|1\rangle := |F = 1, m_F = 0\rangle$$
$$|0\rangle := |F = 0, m_F = 0\rangle$$



$$\hat{H}_{\text{int}} = \frac{1}{2}\left(\Omega_r\,\hat{a}\,\hat{\sigma}^{+}e^{i\delta_r t} + \Omega_b\,\hat{a}^{\dagger}\,\hat{\sigma}^{+}e^{i\delta_b t}\right) + \text{H.c.}$$

$$\hat{H}_{\text{eff}} = \frac{\Omega_b + \Omega_r}{4}\hat{\sigma}_z + \frac{\Omega_b - \Omega_r}{2}\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\left(\Omega_r\,\hat{a} + \Omega_b\,\hat{a}^{\dagger}\right)\hat{\sigma}_x + \text{H.c.}$$

Asymmetric Rabi Hamiltonian

$$\hat{H}_{GR} = \omega \hat{a}^\dagger \hat{a} + \frac{1}{2}\Omega \hat{\sigma}^z + g_1(\hat{a}^\dagger \hat{\sigma}^- + \hat{a}\hat{\sigma}^+) + g_2(\hat{a}^\dagger \hat{\sigma}^+ + \hat{a}\hat{\sigma}^-)$$

Erlingsson et al. (PRB, 2010), Schliemann et al. (PRB, 2003)
Tomka et al. (PRA, 2014), Schen et al. (APB, 2014), …

Asymmetric Rabi Hamiltonian

(Mostly at Resonance)

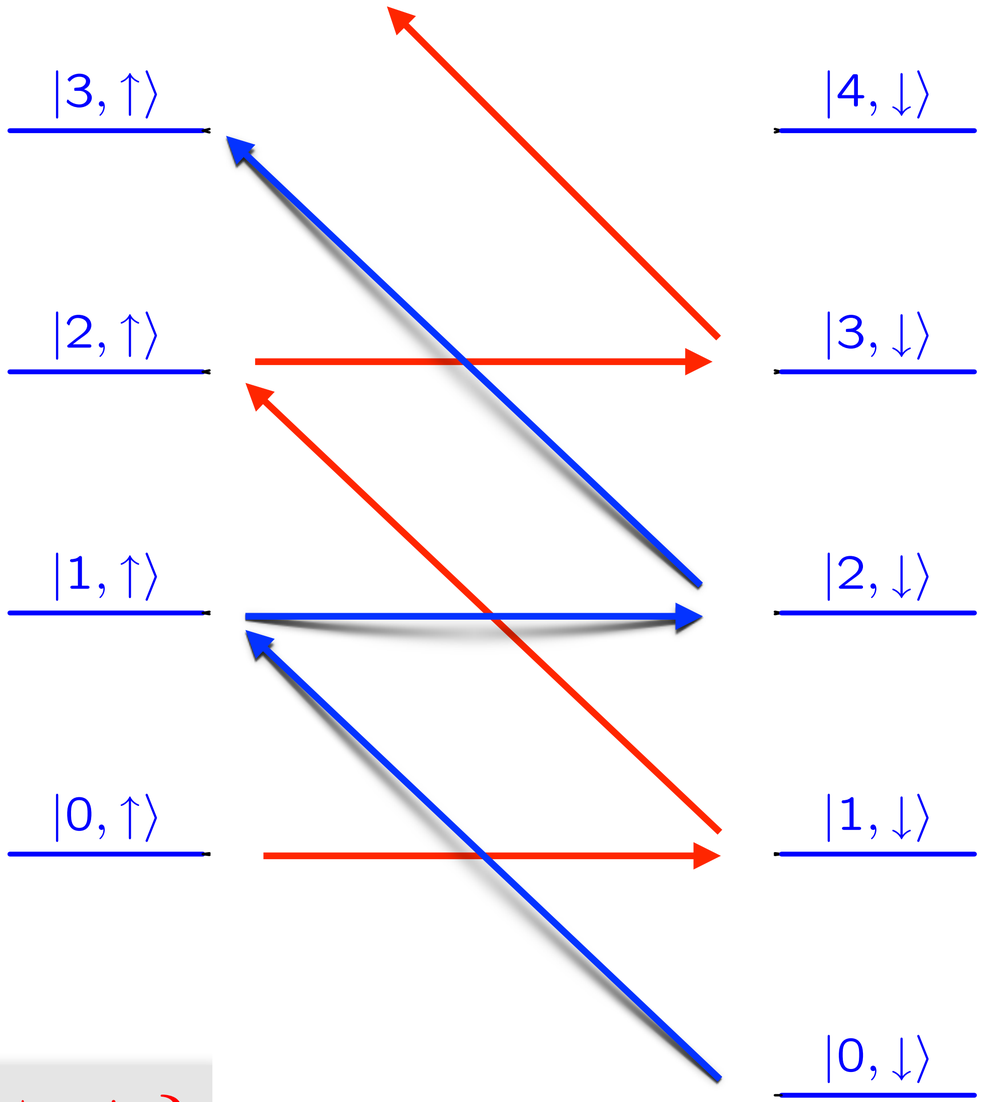
$$\hat{H}_{GR} = \omega_c \hat{a} + \frac{1}{2}\Omega \hat{\sigma}^z + g_1(\hat{a}^\dagger \hat{\sigma}^- + \hat{a}\hat{\sigma}^+) + g_2(\hat{a}^\dagger \hat{\sigma}^+ + \hat{a}\hat{\sigma}^-)$$

Erlingsson et al. (PRB, 2010), Schliemann et al. (PRB, 2003)
Tomka et al. (PRA, 2014), Schen et al. (APB, 2014), …

Parity Symmetry

$$\hat{\Pi} := e^{i\pi\hat{n}}\hat{\sigma}^z$$

$$[\hat{H}, \hat{\Pi}] = 0$$

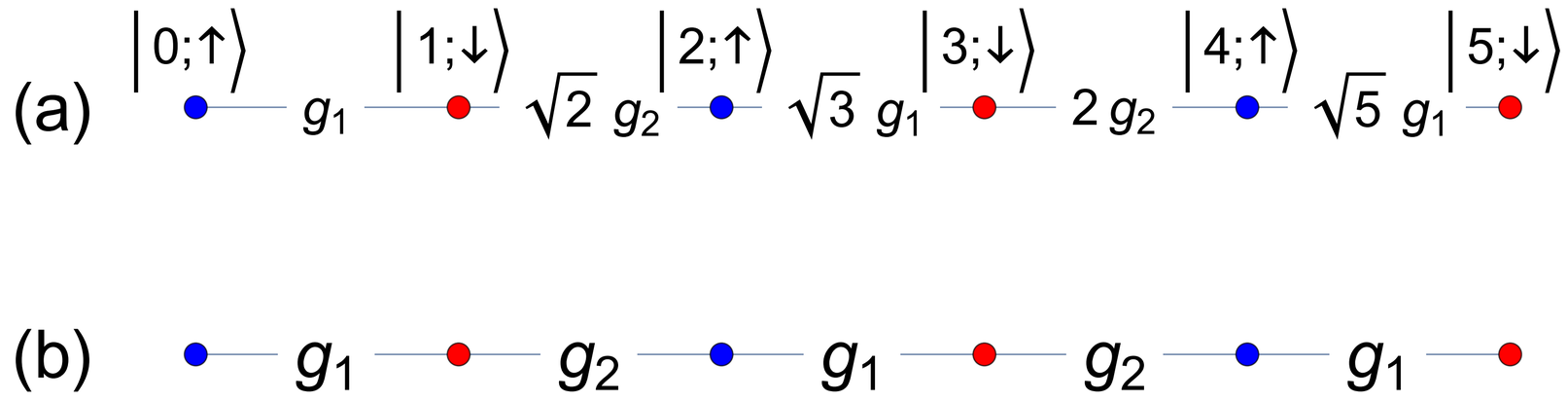


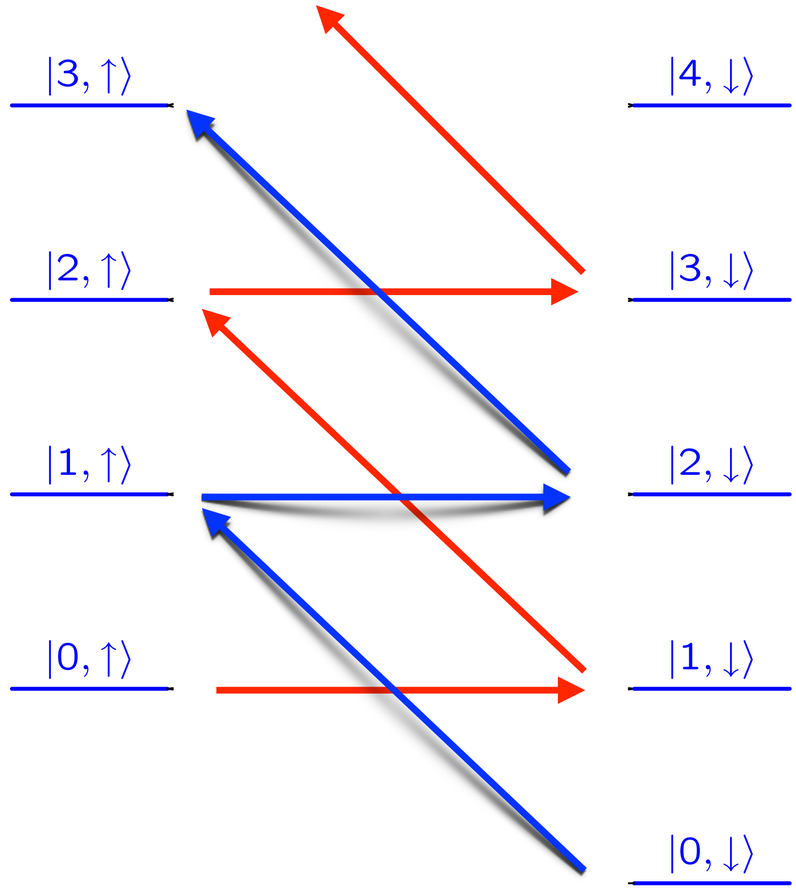
$$\{\, |n,\sigma\rangle \mid n = 0,1,2,\cdots;\ \sigma = \uparrow,\downarrow \,\}$$

Topological Properties

Chiral Symmetry

(Non-uniform Chain)



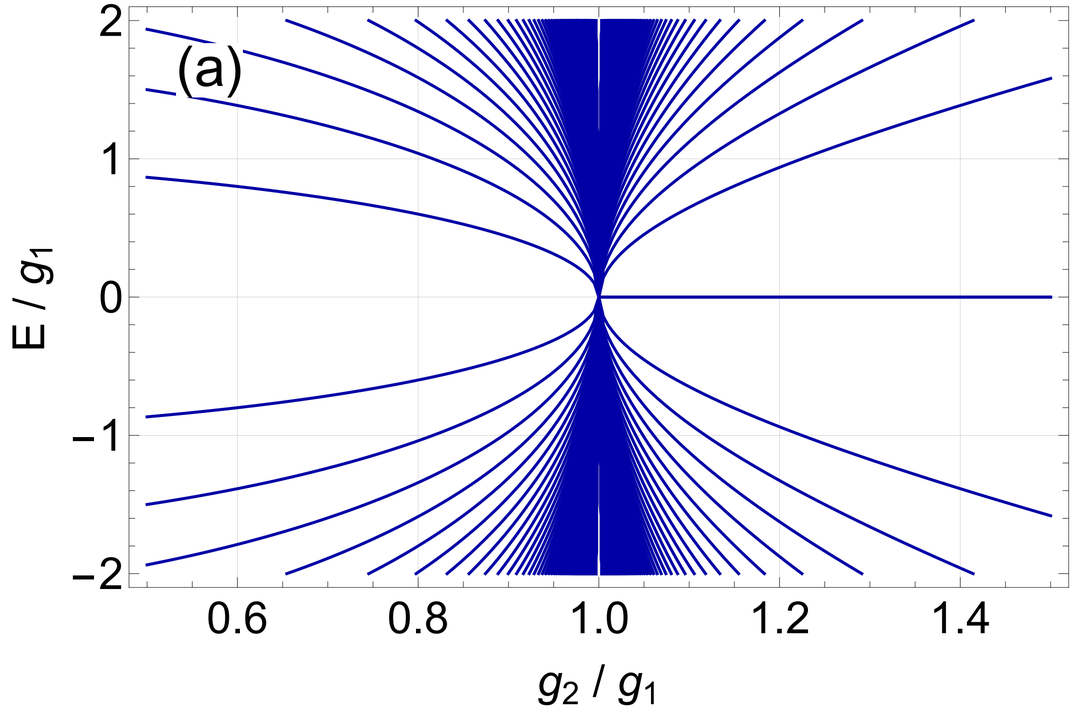


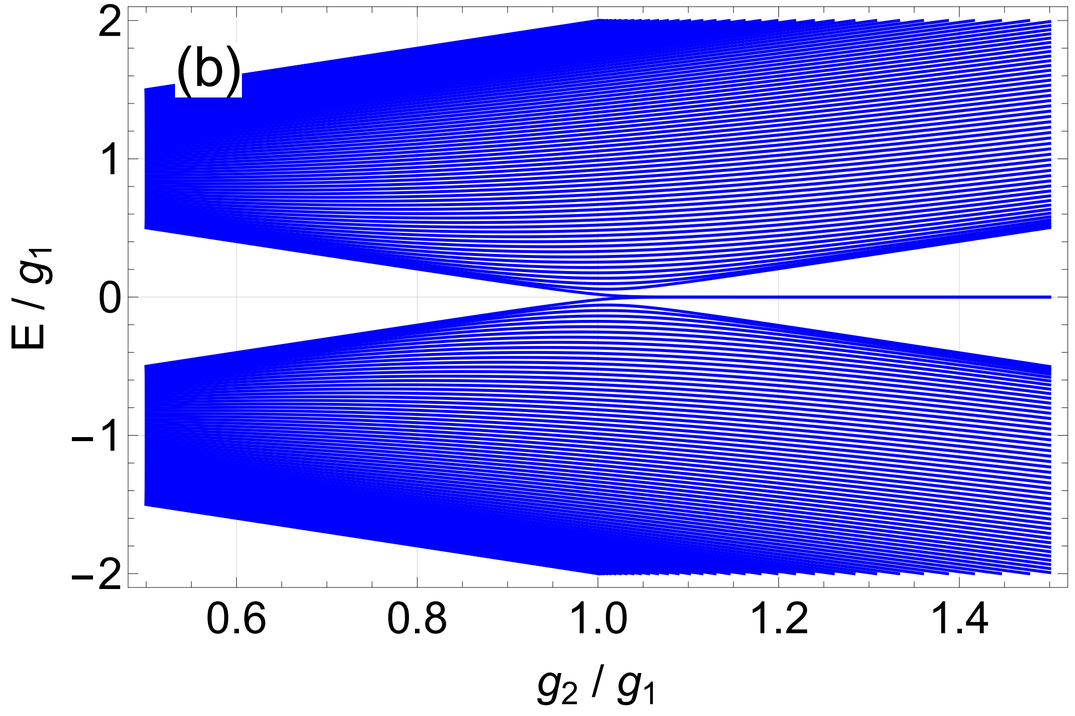
$$\Xi_e := e^{i\pi\hat{n}} = \hat{\sigma}^z$$

$$\{\hat{H}_e, \hat{\Xi}_e\} = 0$$

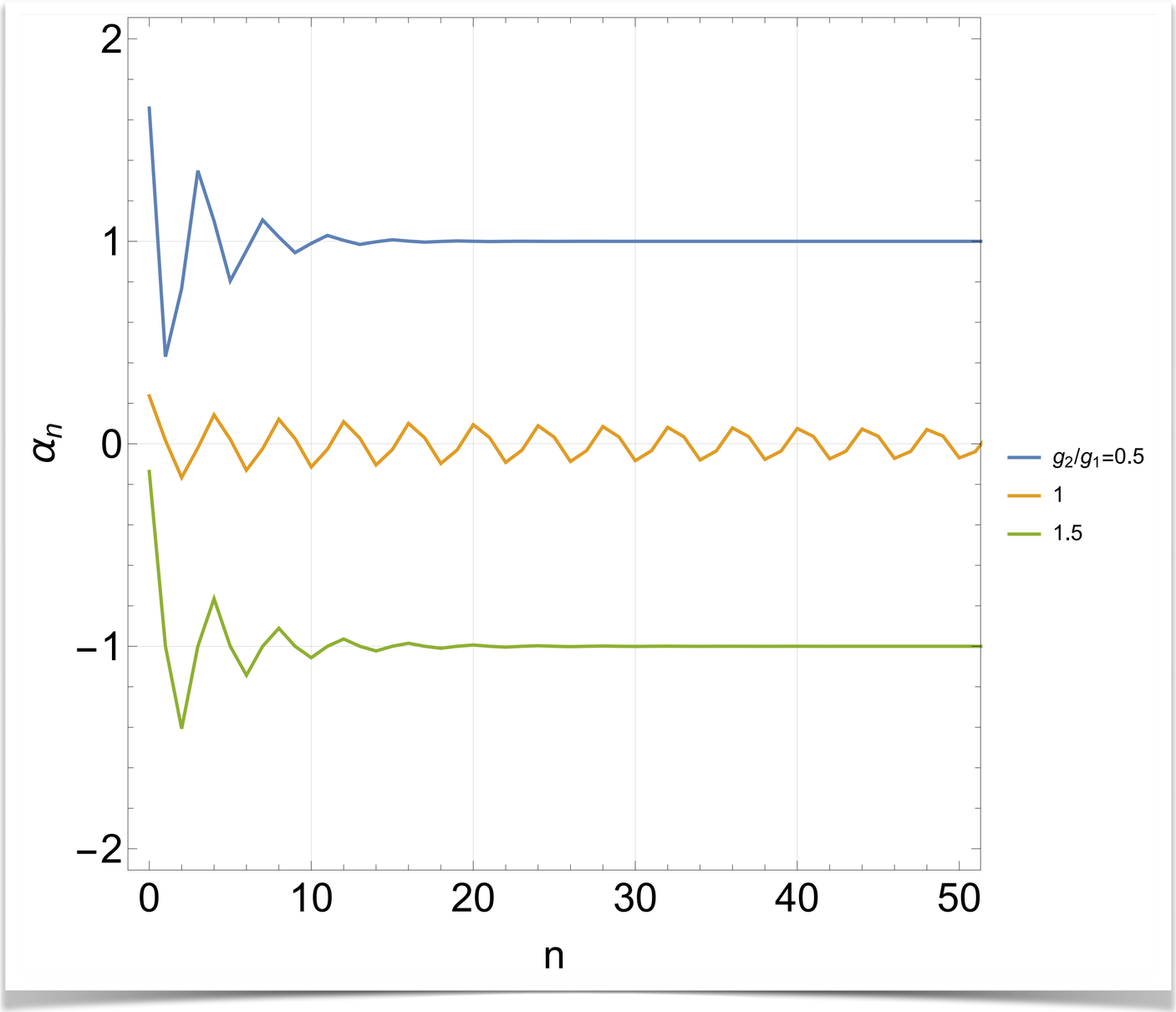
Topological Transition

Zero-Energy "Edge" State





THE 'GROUND STATES'



The Zero-Energy State

Squeezed & Separable State

$$|D\rangle := \underbrace{\hat{S}^\dagger(\theta/2)\,|0\rangle}_{\text{photon}} \otimes \overbrace{|\uparrow\rangle}^{\text{qubit}}$$

$$\hat{S}(z) := \exp\left(\frac{1}{2} z \hat{a}^\dagger \hat{a}^\dagger - h.c.\right)$$

$$\cosh\theta := \frac{g_1^2 + g_2^2}{|g_1^2 - g_2^2|}$$

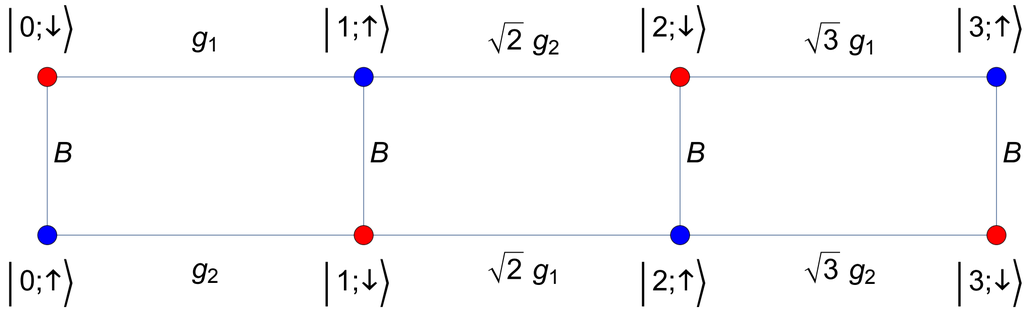
$$\sinh\theta := \frac{2 g_1 g_2}{|g_1^2 - g_2^2|}$$

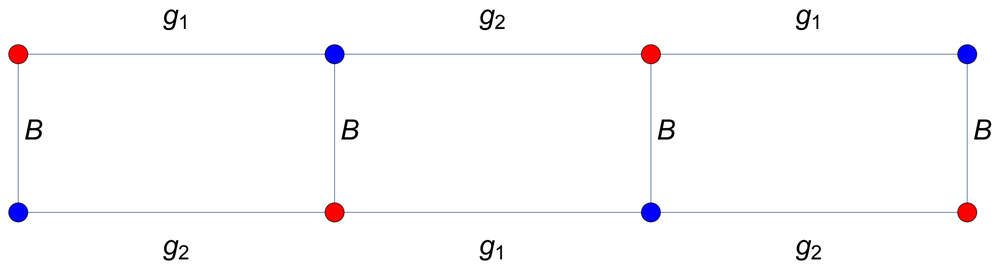
Parity Mixing I

By Applying External magnetic Field on Spin

$$\hat{H}_{\text{ext}} \sim B\hat{\sigma}^x = B(\hat{\sigma}^+ + \hat{\sigma}^-)$$

Chiral Symmetry



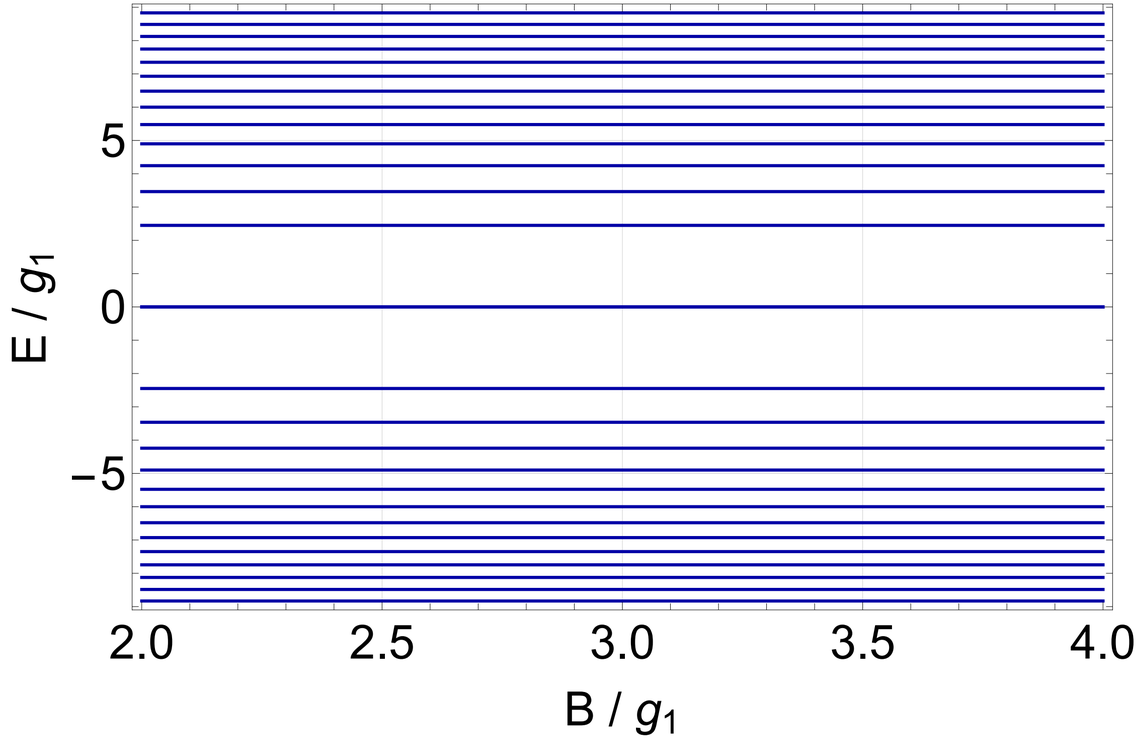


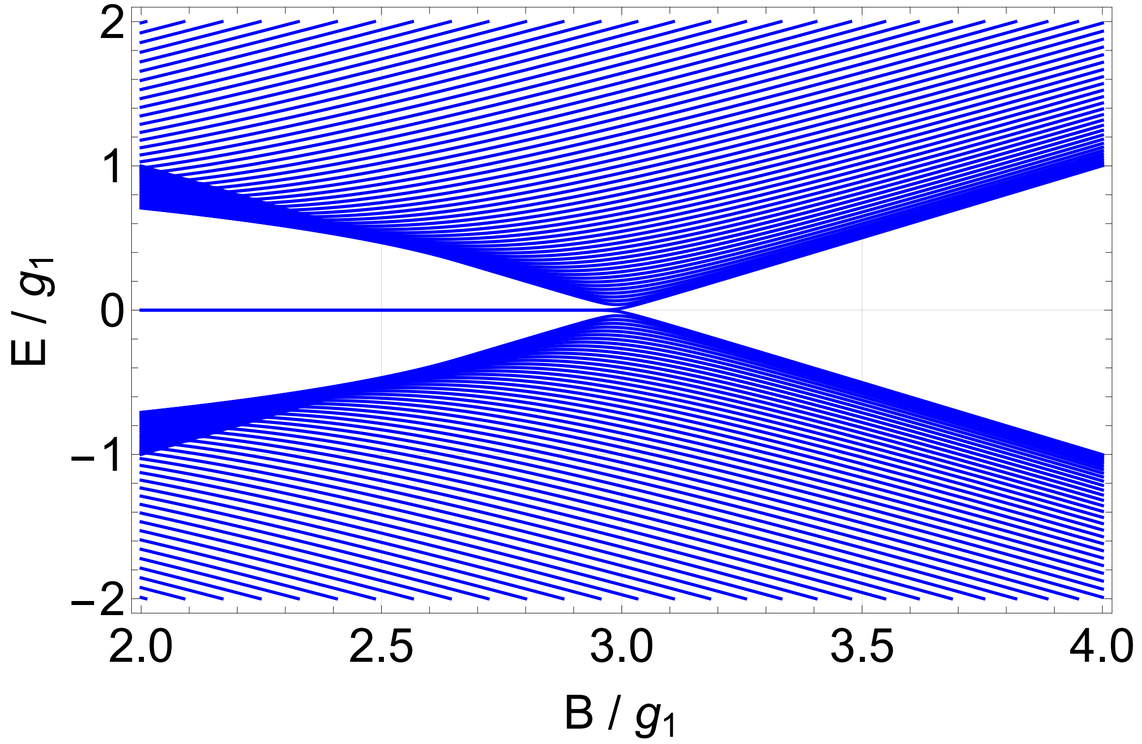
$$\hat{H} = (g_1\hat{a} + g_2\hat{a}^\dagger + B)\hat{\sigma}^+ + (g_2\hat{a} + g_1\hat{a}^\dagger + B)\hat{\sigma}^-$$

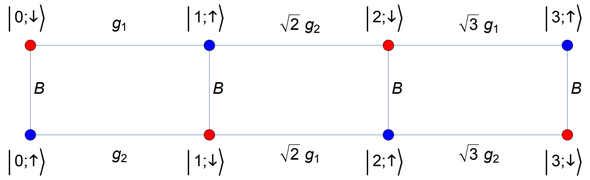
$$\hat{\Xi} = \hat{\sigma}^z$$

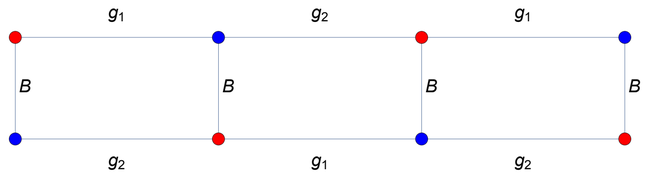
No Transition

Always Topologically Non-trivial









The Zeo-Energy State

Squeezed & Separable State

$$|D\rangle = \hat{D}^\dagger(\xi)\hat{S}^\dagger(\theta/2)\,|0\rangle \otimes |\downarrow / \uparrow\rangle$$

$$\hat{S}(z) := \exp\left(\frac{1}{2}z\hat{a}^\dagger\hat{a}^\dagger - h.c.\right)$$

$$\hat{D}(z) := \exp(z\hat{a}^\dagger - z^*\hat{a})$$

$$\cosh\theta := \frac{g_1^2 + g_2^2}{|g_1^2 - g_2^2|}$$

$$\sinh\theta := \frac{2g_1g_2}{|g_1^2 - g_2^2|}$$

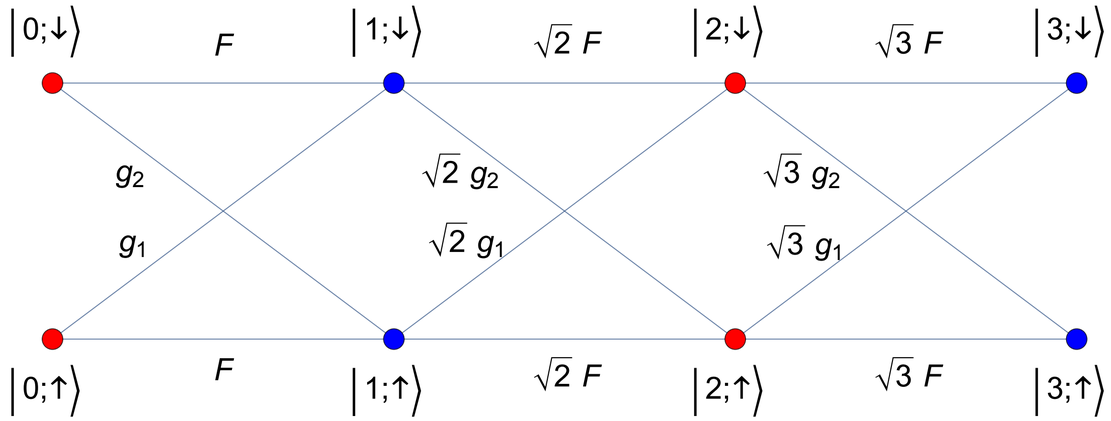
$$\xi := \frac{B}{g_1 + g_2}$$

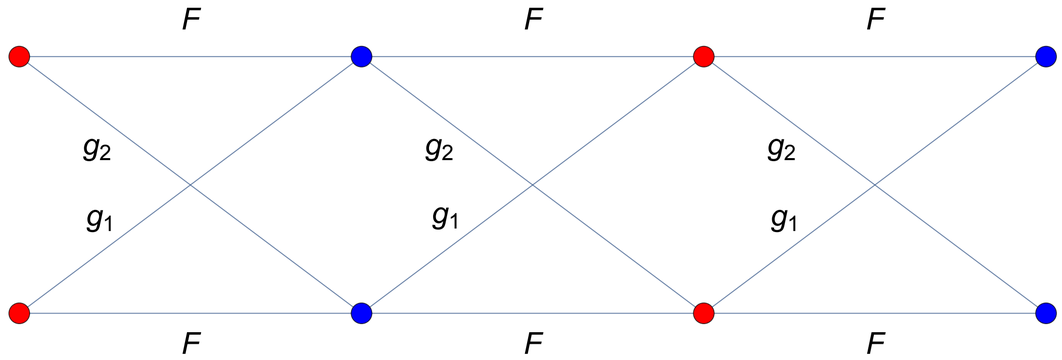
Parity Mixing II

By Applying Cavity Input Field

$$\hat{H}_{\mathrm{ext}} \sim F(\hat{a}^{\dagger} + \hat{a})$$

Chiral Symmetry



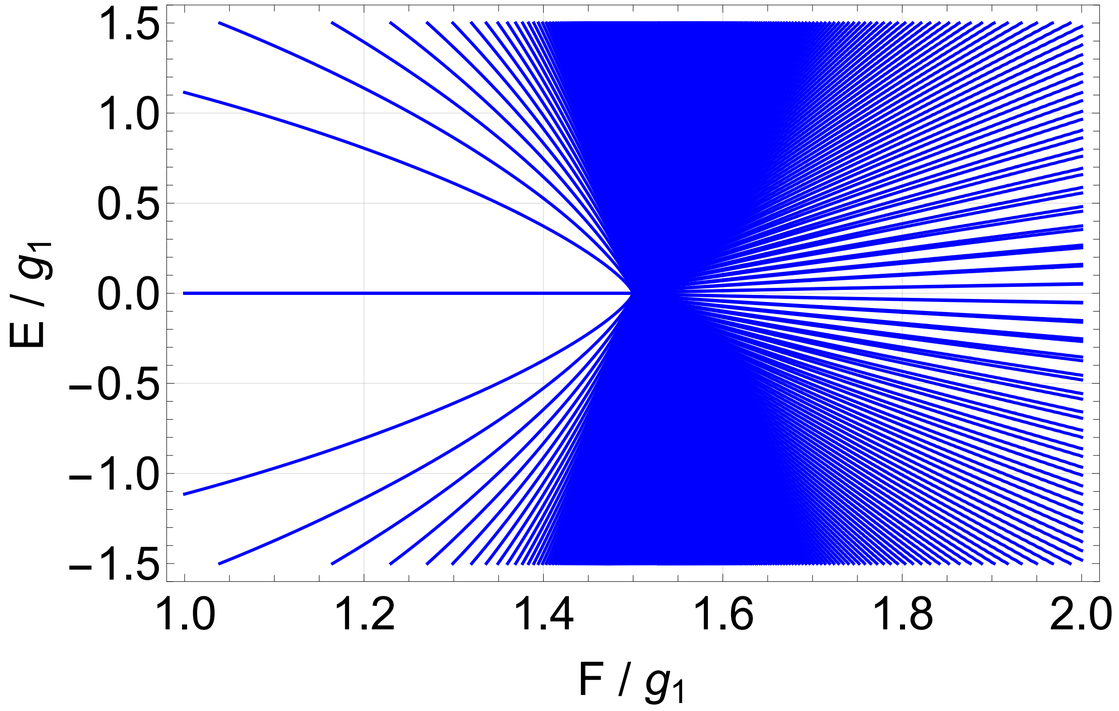


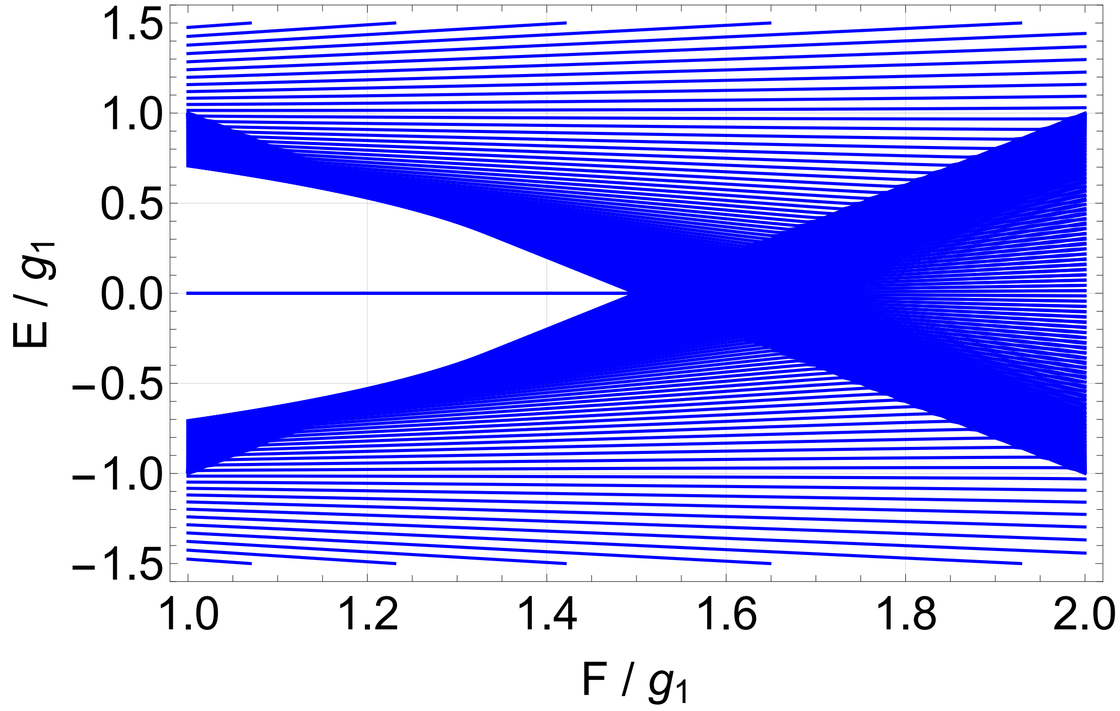
$$\hat{H} = \hat{a}\left(g_1\hat{\sigma}^+ + g_2\hat{\sigma}^- + F\right) + h.c.$$

$$\Xi = e^{i\pi\hat{n}}$$

Topological Transition

From Topological Insulator to Weyl Metal





$$(g_1 - g_2)\left[(g_1 + g_2)^2 - 4F^2\right] > 0$$

Zero-Energy "Edge" State

Squeezed & Separable State

$$|D\rangle = \hat{S}^\dagger(\theta)\,|0\rangle \otimes (|\uparrow\rangle \cos\phi + |\downarrow\rangle \sin\phi)$$

$$\tanh\theta = \Gamma := \operatorname{sign} x \left(|x| - \sqrt{x^2 - 1}\right)$$

$$\tan\phi := \frac{F(1-\Gamma)}{g_1\Gamma - g_2} = \frac{g_2\Gamma - g_1}{F(1-\Gamma)}$$

$$x := \frac{F^2 - (g_1^2 + g_2^2)/2}{F^2 - g_1 g_2}$$

Various Topological Transitions

From topological insulator to Weyl semi-metal

		GRD	Regularized
$N=1$	$\mathcal{H}_e$	I / TI	I / TI
	$\mathcal{H}_o$	TI / I	TI / I
	$B \neq 0$	TI (always)	TI / I
	$F \neq 0$	TI / M	TI / M
$N=2$	$\mathcal{H}_e$	TI / WSM	TI / WSM
	$\mathcal{H}_o$	TI / WSM	TI / WSM
	$B_j \neq 0$	I / WSM	I / WSM
	$F \neq 0$	TI / M	TI / M

Summary

- Generalized Rabi Hamiltonian
- Possible for circuit QED systems with parametric control of coupling?
- Various topological transitions with external fields.
- Zero-energy "edge" states show squeezing and separable from spins (qubits).