QChips 2026 (Gdansk, Poland, August 26, 2026)

Ultra-Strong Coupling Physics in Circuit-QED Systems

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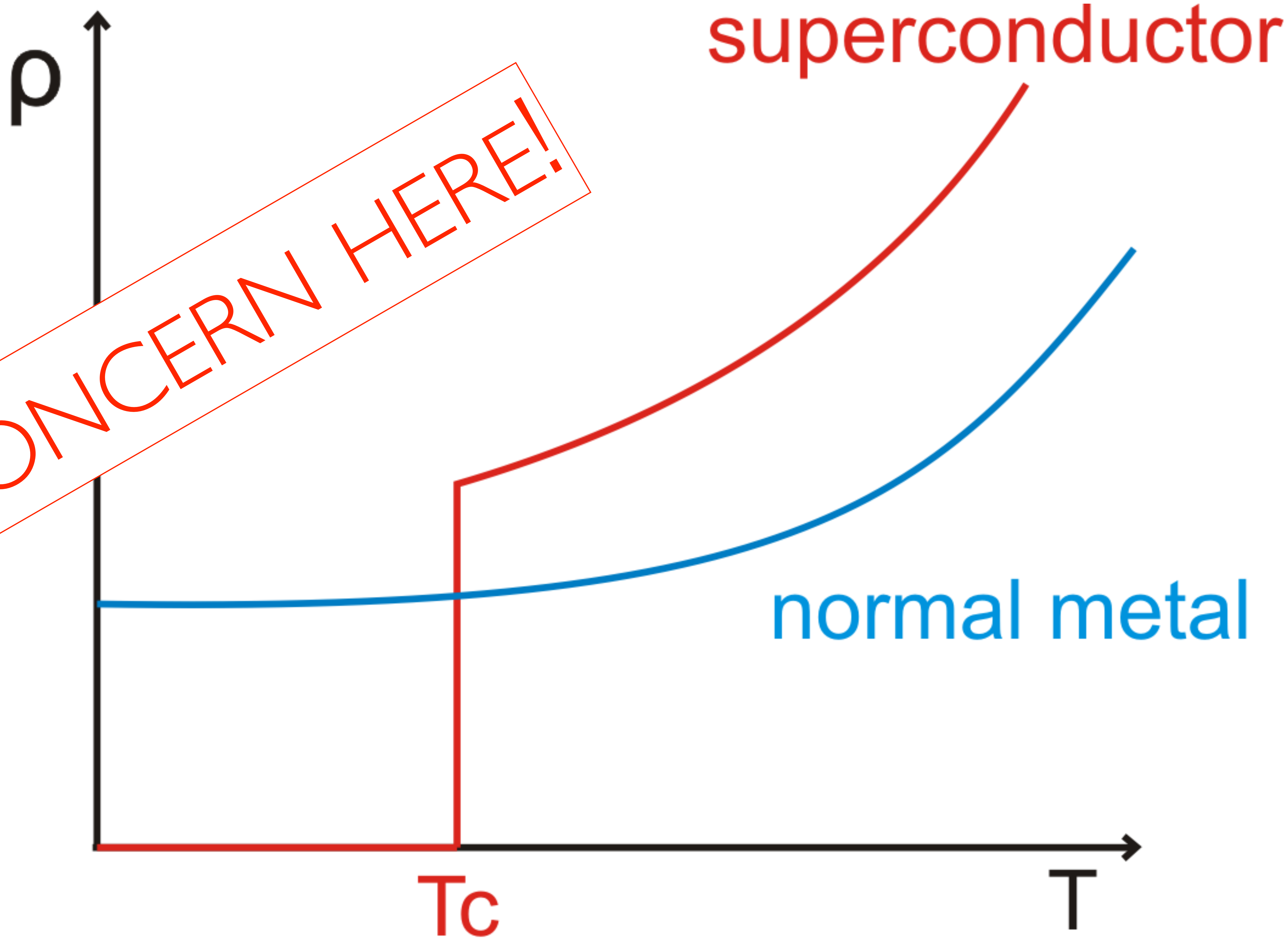
(Korea University)

Superconductivity

Phenomena

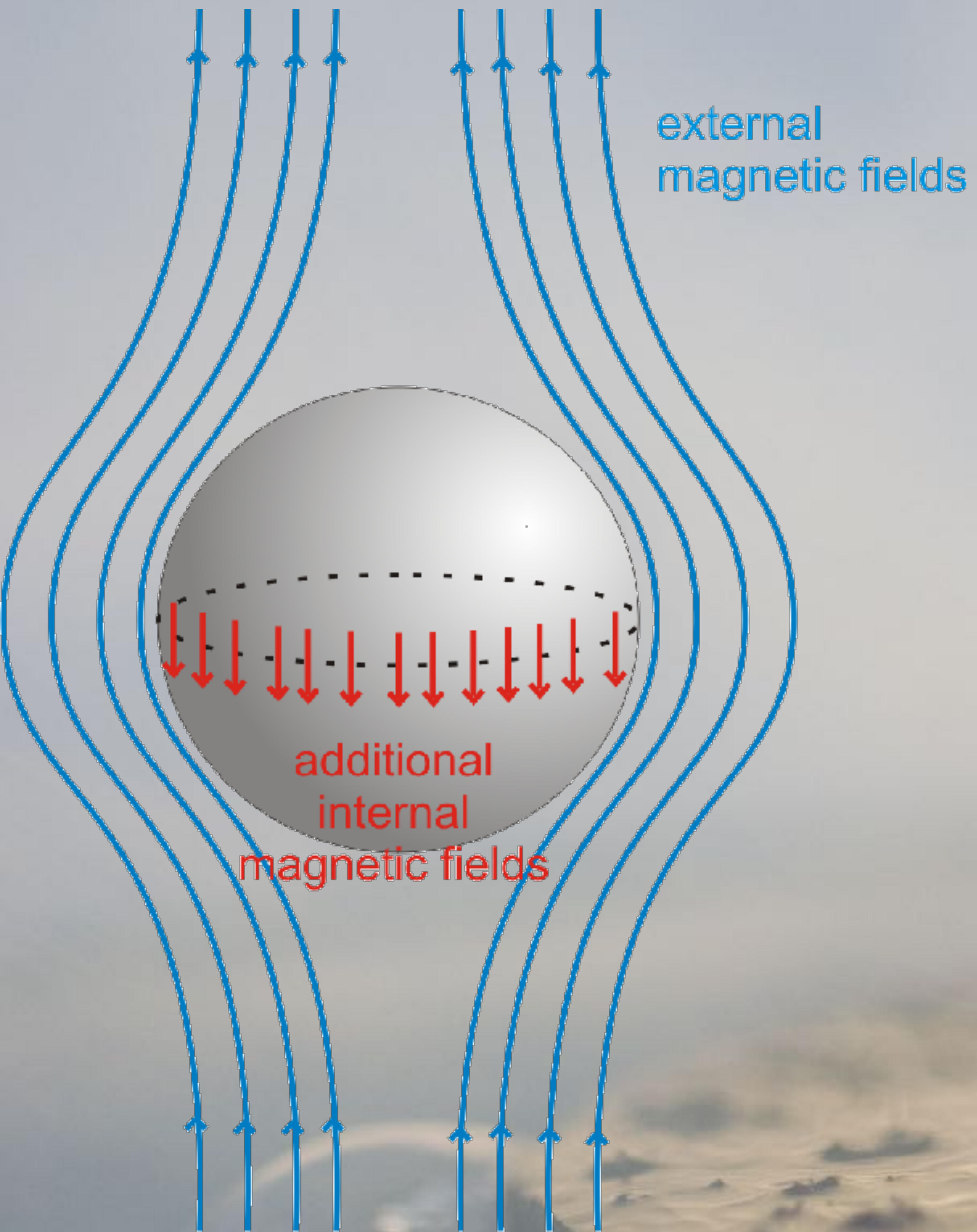
Perfect Conductivity



Not a concern here!

Perfect Diamagnetism

Meissner Effect



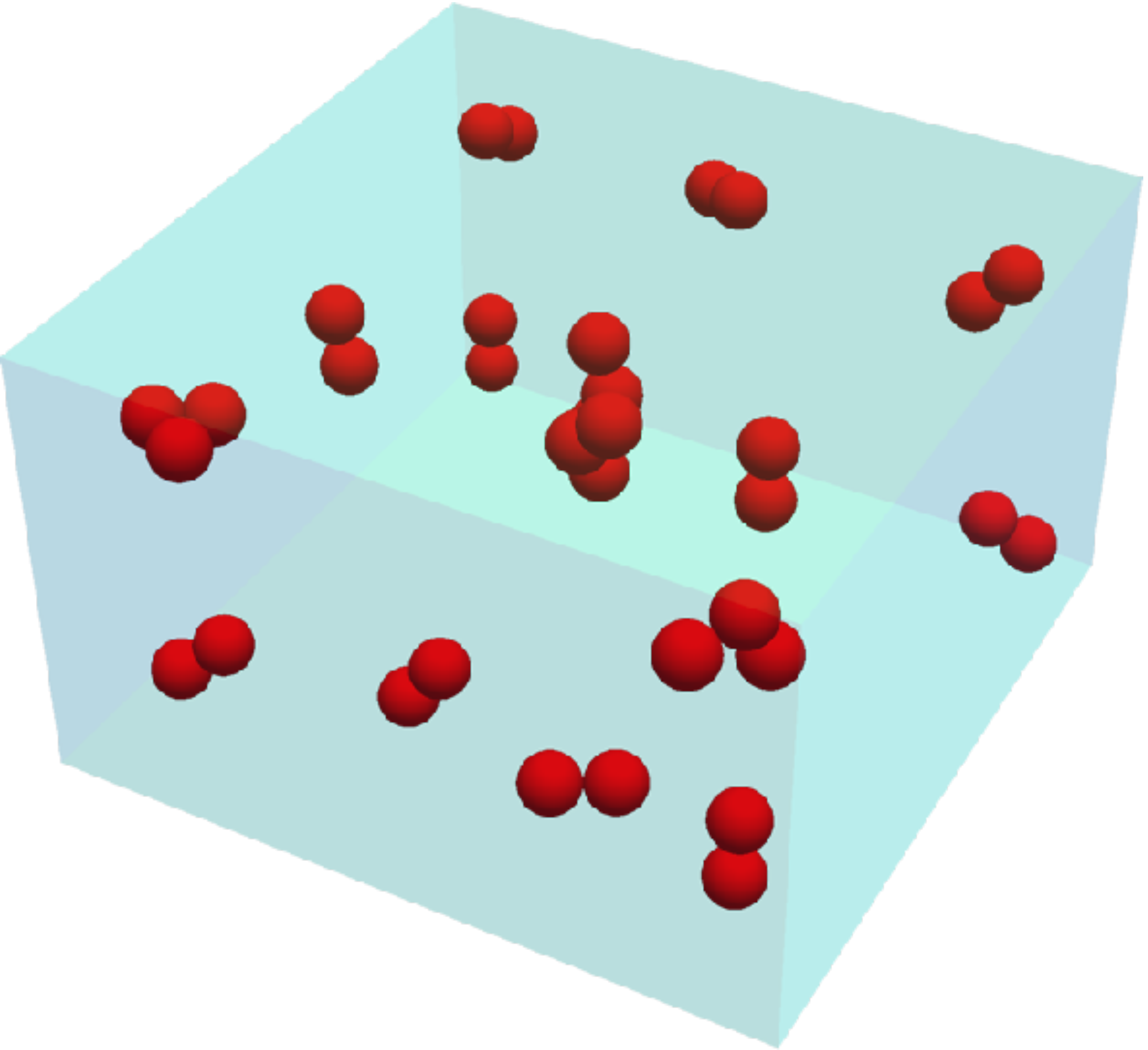
More Relevant

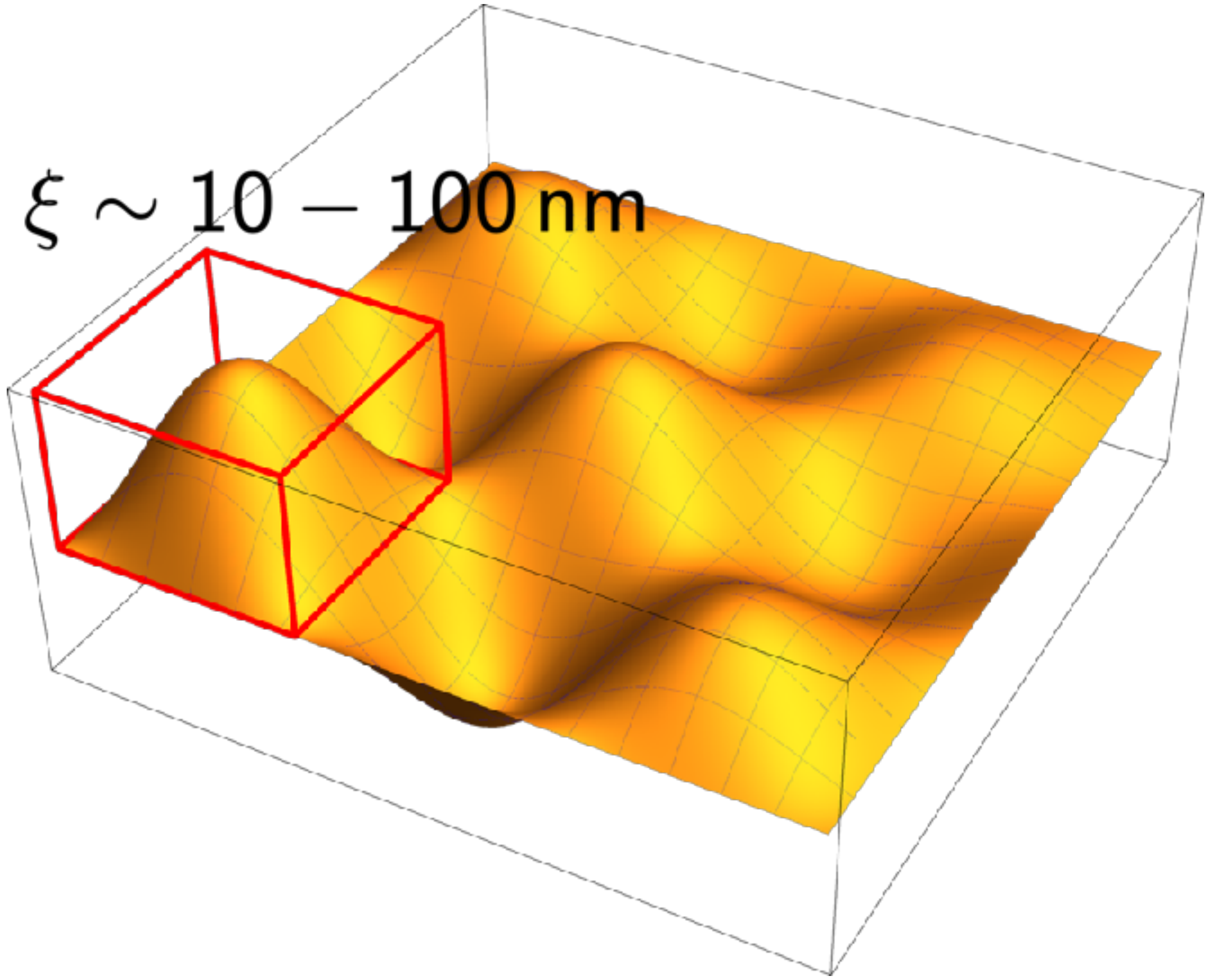
SUPERCONDUCTIVITY

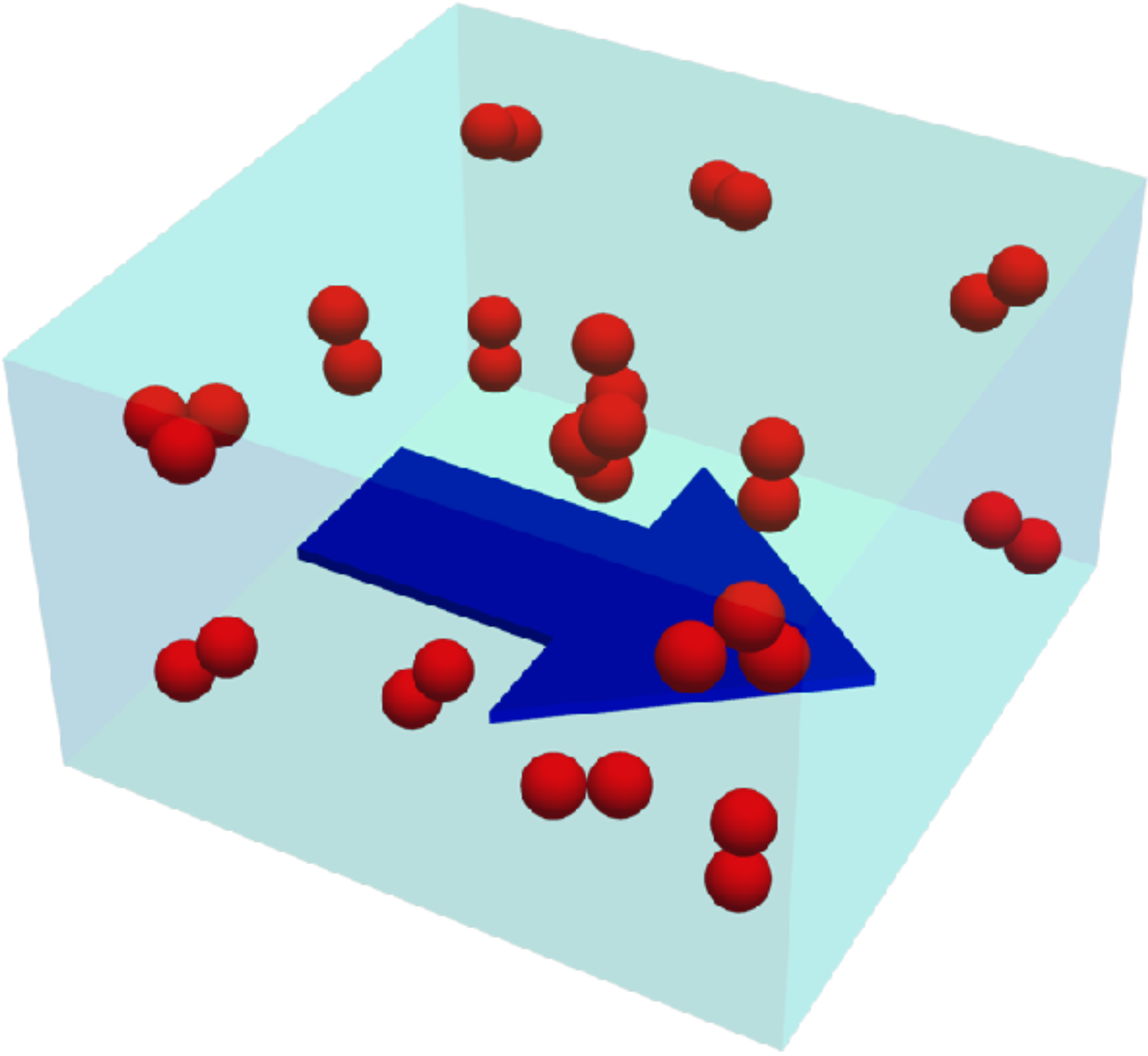
THEORY

The Ginzburg-Landau Theory

Complex Order Parameter





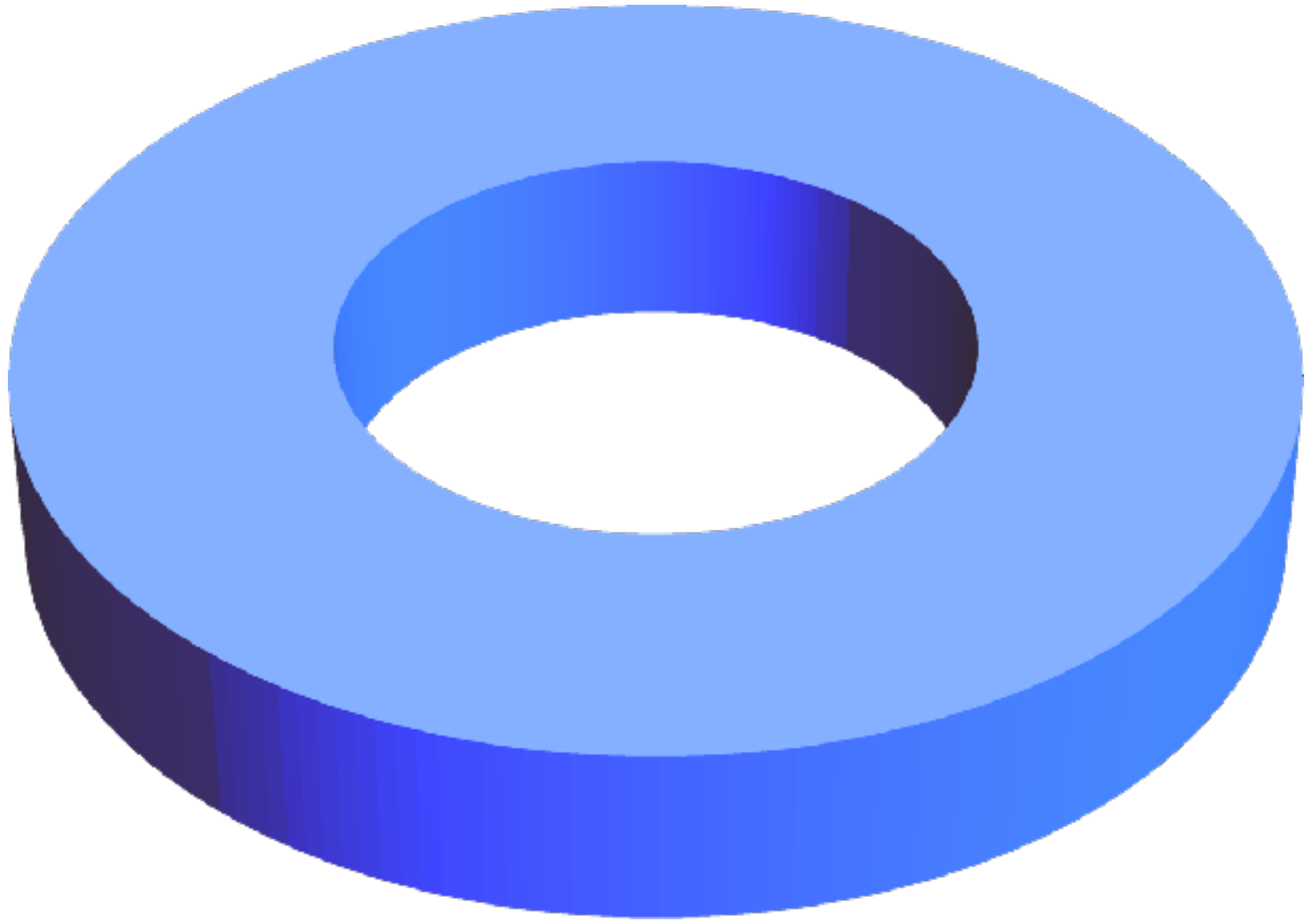


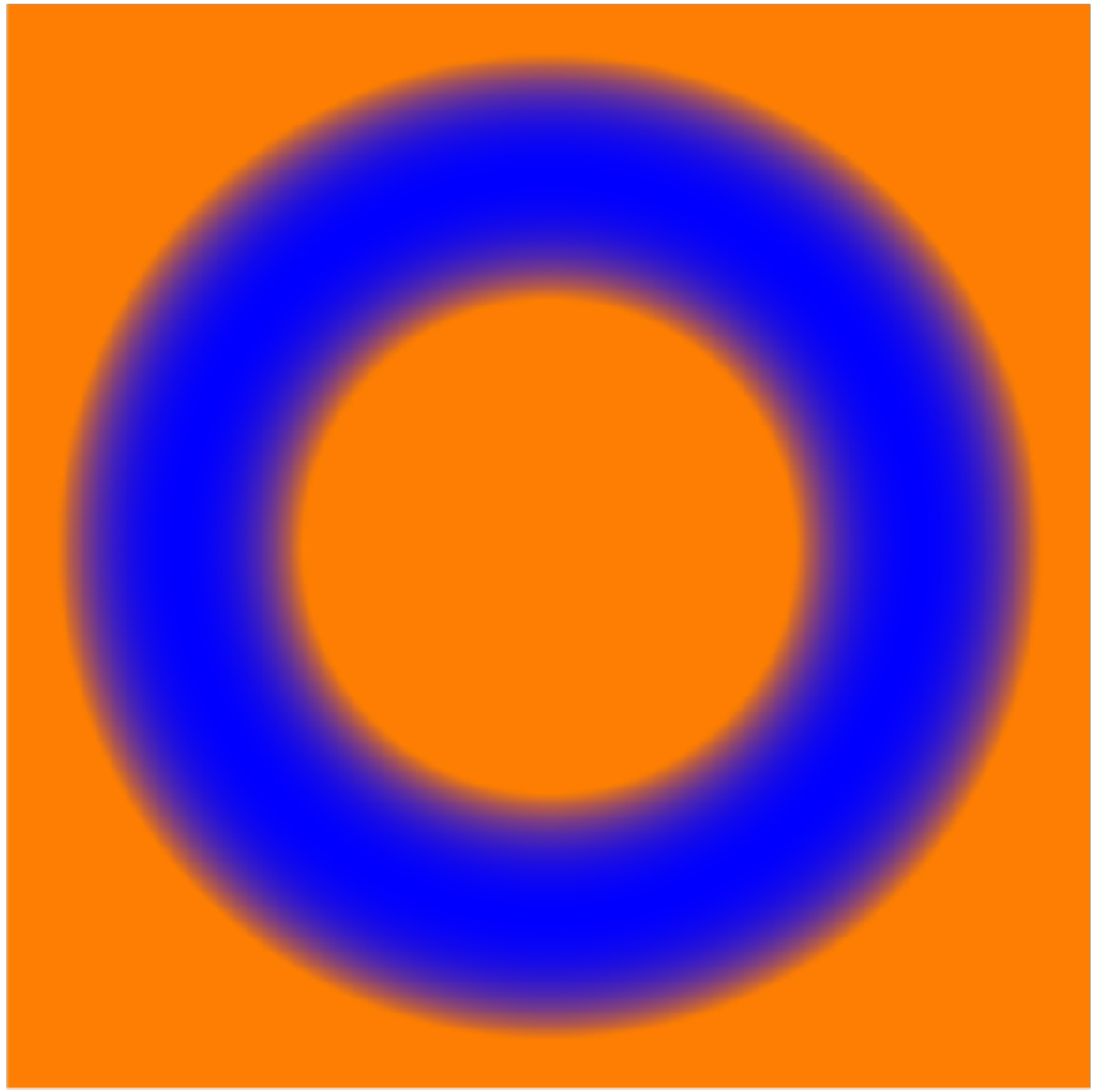
$$\Psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\exp[i\phi(\boldsymbol{r})]$$

$$\boldsymbol{J}_S(\boldsymbol{r}) \propto \boldsymbol{\nabla}\phi + 2\pi\boldsymbol{A}/\Phi_0$$

Flux Quantization

(Quantized Super-Current)





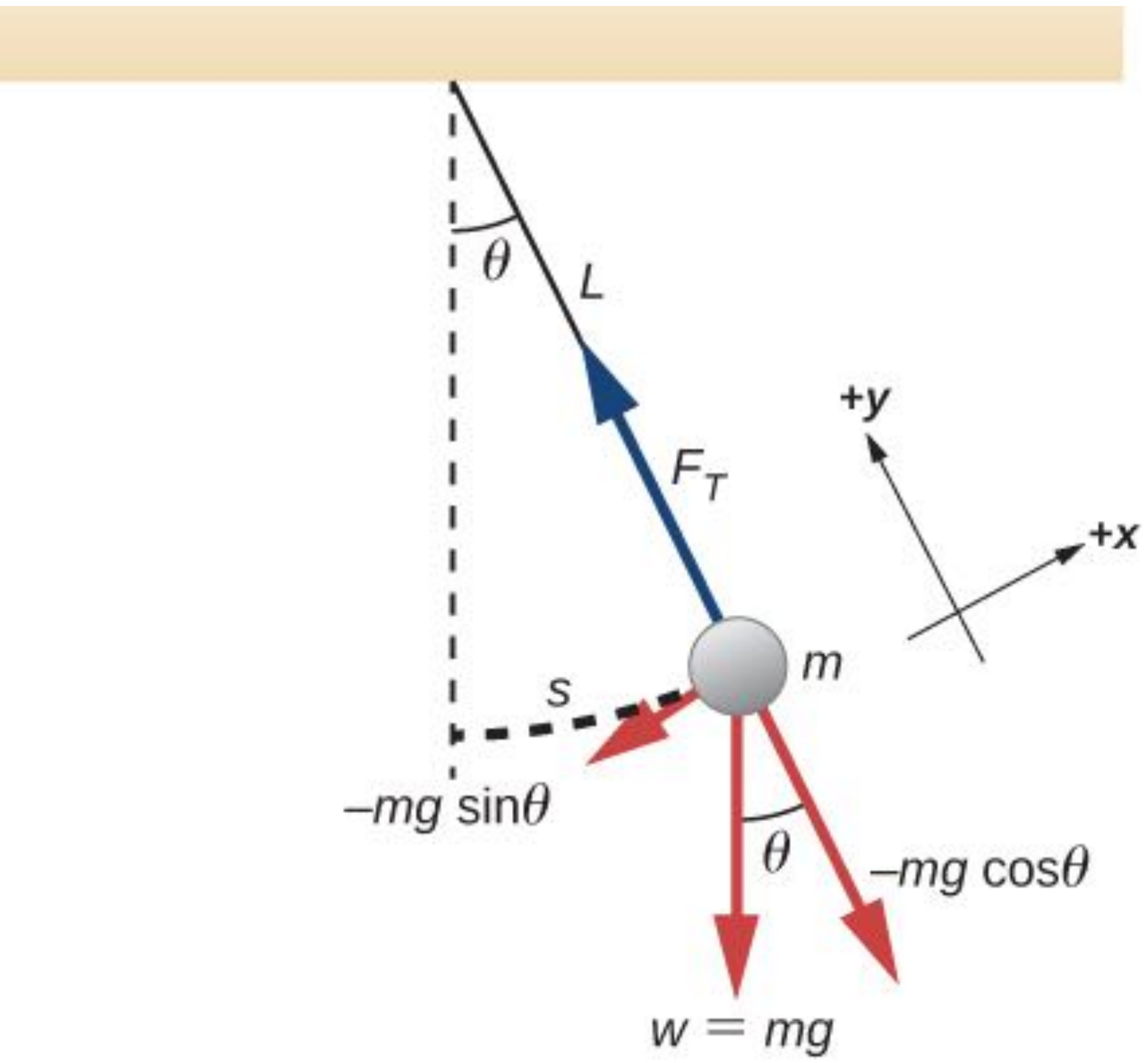
Single-valued-ness

$$\frac{\Phi}{\Phi_0} \in \mathbb{Z}$$

$$\Phi_0 := \frac{h}{2e}$$

Something very different

Simple Pendulum



$$\boldsymbol{F} = \frac{d\boldsymbol{P}}{dt} = m\boldsymbol{a} = m\ddot{\boldsymbol{r}}$$

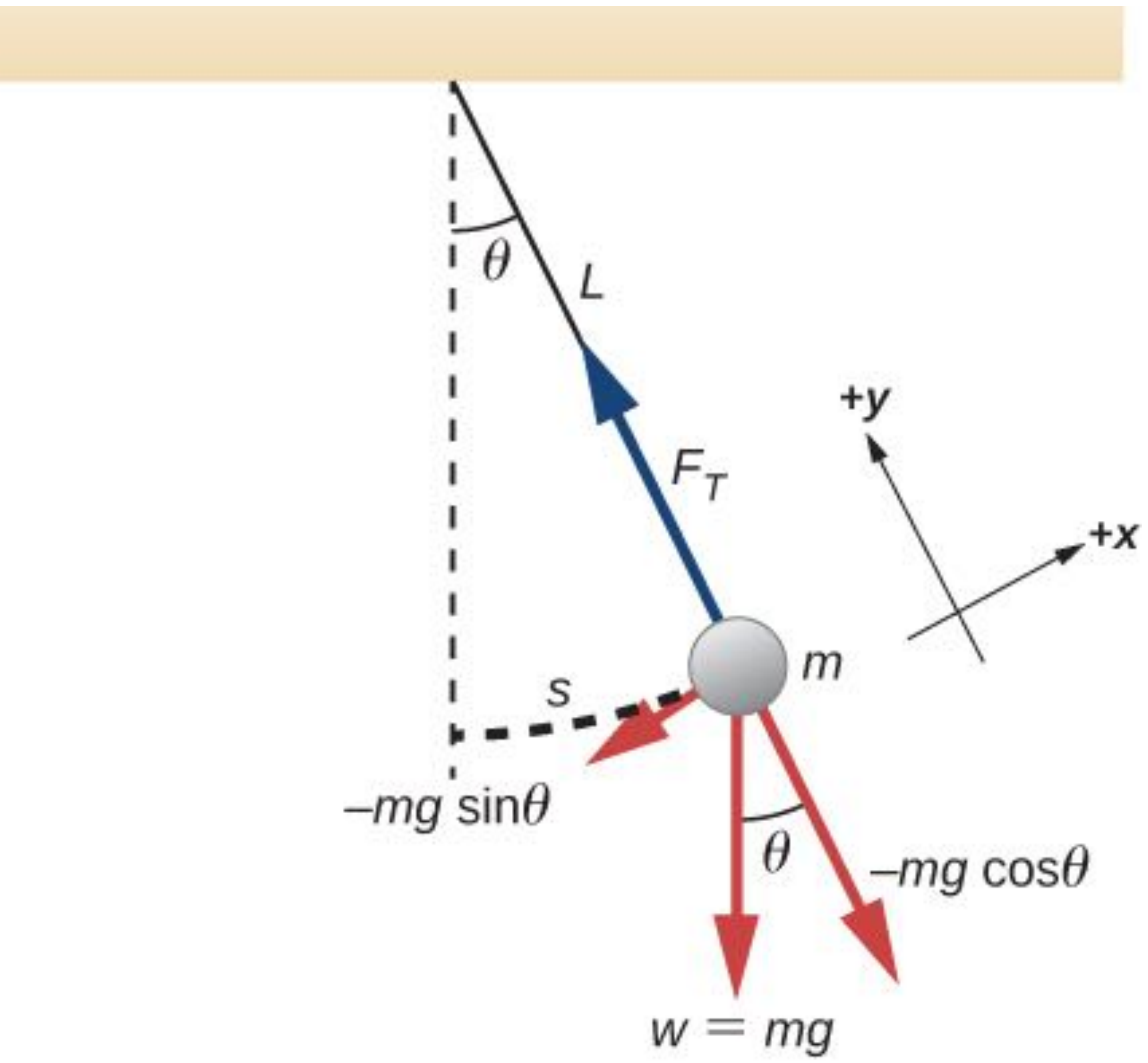
$$\boldsymbol{\tau} = \frac{d\boldsymbol{L}}{dt}$$

$$\boldsymbol{\tau} := \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{L} := \boldsymbol{r} \times \boldsymbol{P}$$

$$-mg\sin\theta = m\ell^2\ddot{\theta}$$

SIMPLE PENDULUM

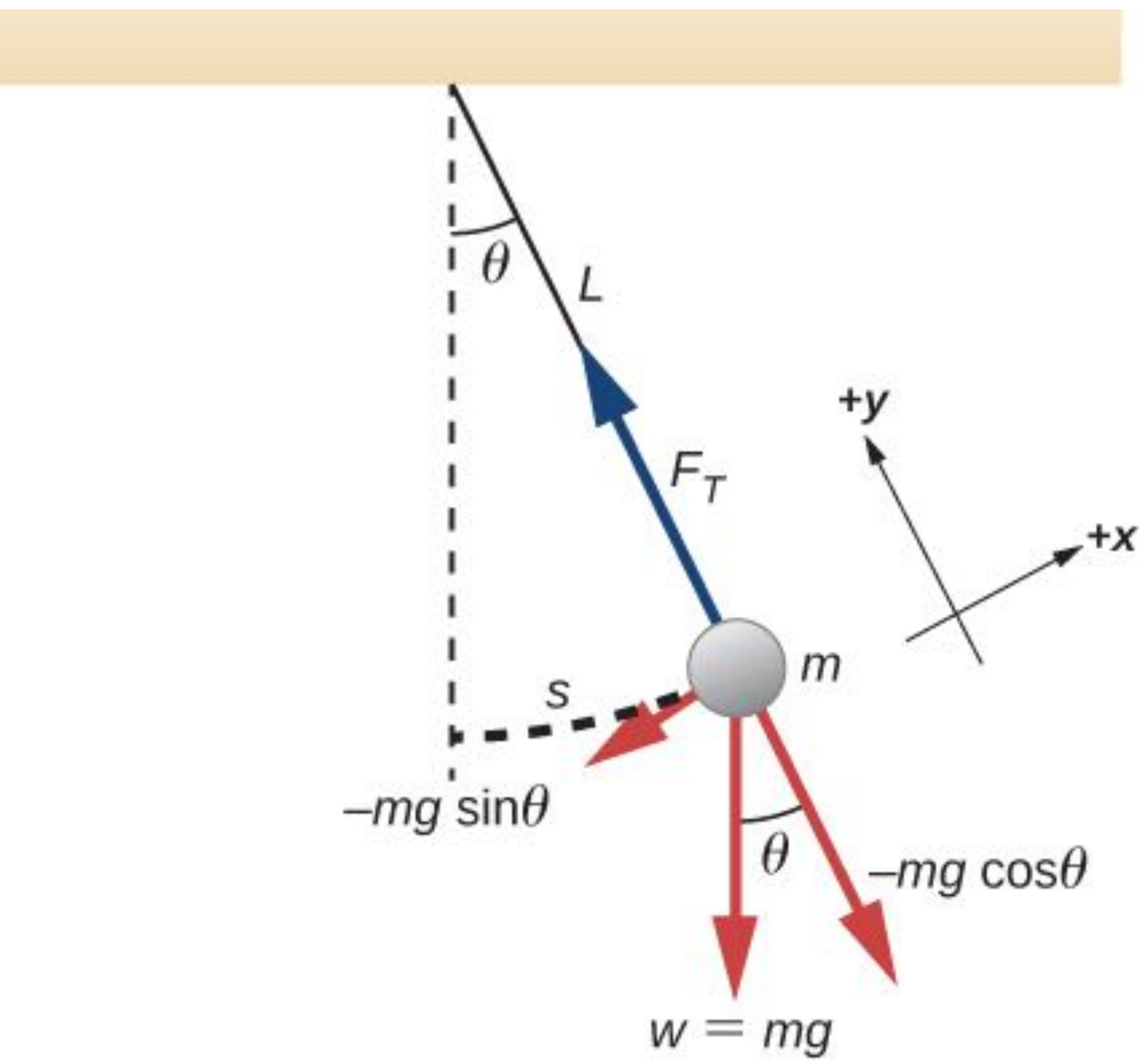


$$\mathscr{L} = T - V = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell(1-\cos\theta)$$

$$\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\theta}} - \frac{\partial \mathscr{L}}{\partial \theta} = 0$$

$$-mg\sin\theta = m\ell^2\ddot{\theta}$$

SIMPLE PENDULUM

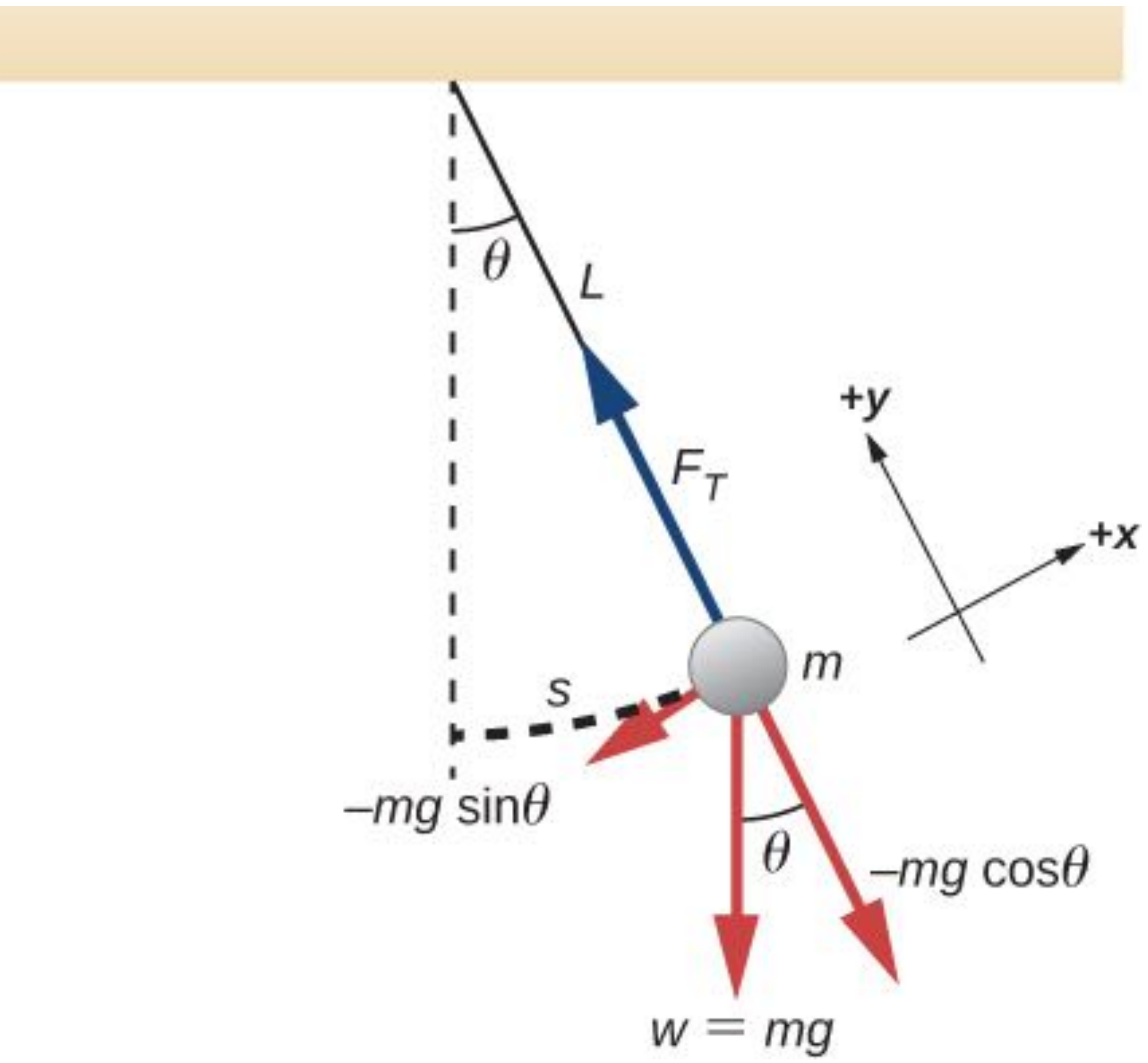


$$H = T + V = \frac{P_\theta^2}{2m\ell^2} + mg\ell(1 - \cos\theta)$$

$$H = P_\theta \dot{\theta} - \mathscr{L}\,, \quad P_\theta := \frac{\partial \mathscr{L}}{\partial \dot{\theta}}$$

$$\{\theta, P_\theta\} = 1$$

Quantum Pendulum

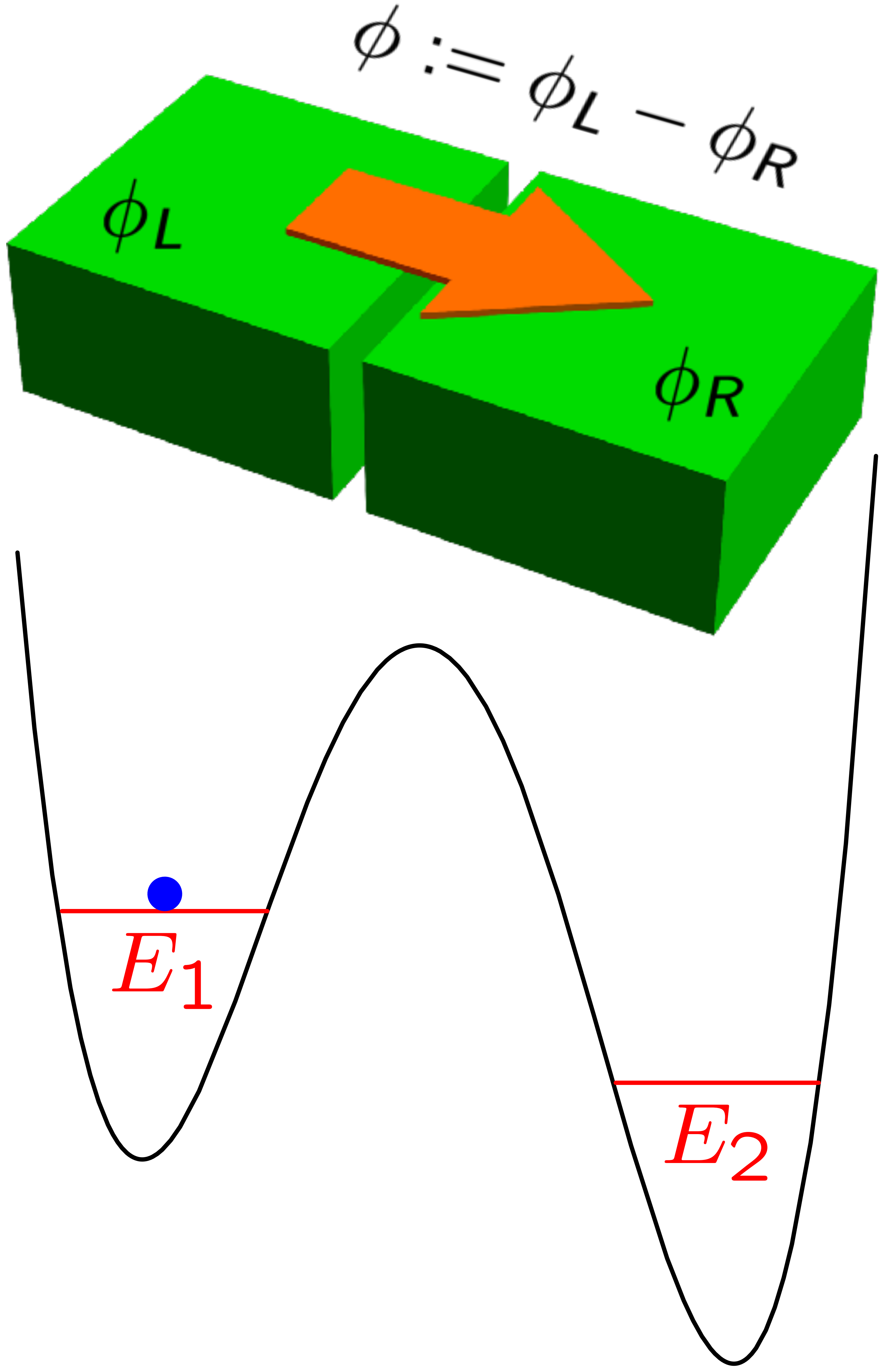


$$\hat{H} = \frac{\hat{P}_\theta^2}{2m\ell^2} - mg\ell\cos\hat{\theta}$$

$$[\hat{\theta}, \hat{P}_\theta] = i\hbar$$

Josephson Effects

Josephson Junctions



$$i\hbar\dot{\Psi}_L = E_L\Psi_L - J\Psi_R\,, \quad \Psi_L = \sqrt{N_L}e^{i\phi_L}$$
$$i\hbar\dot{\Psi}_R = E_R\Psi_R - J\Psi_L\,, \quad \Psi_R = \sqrt{N_R}e^{i\phi_R}$$

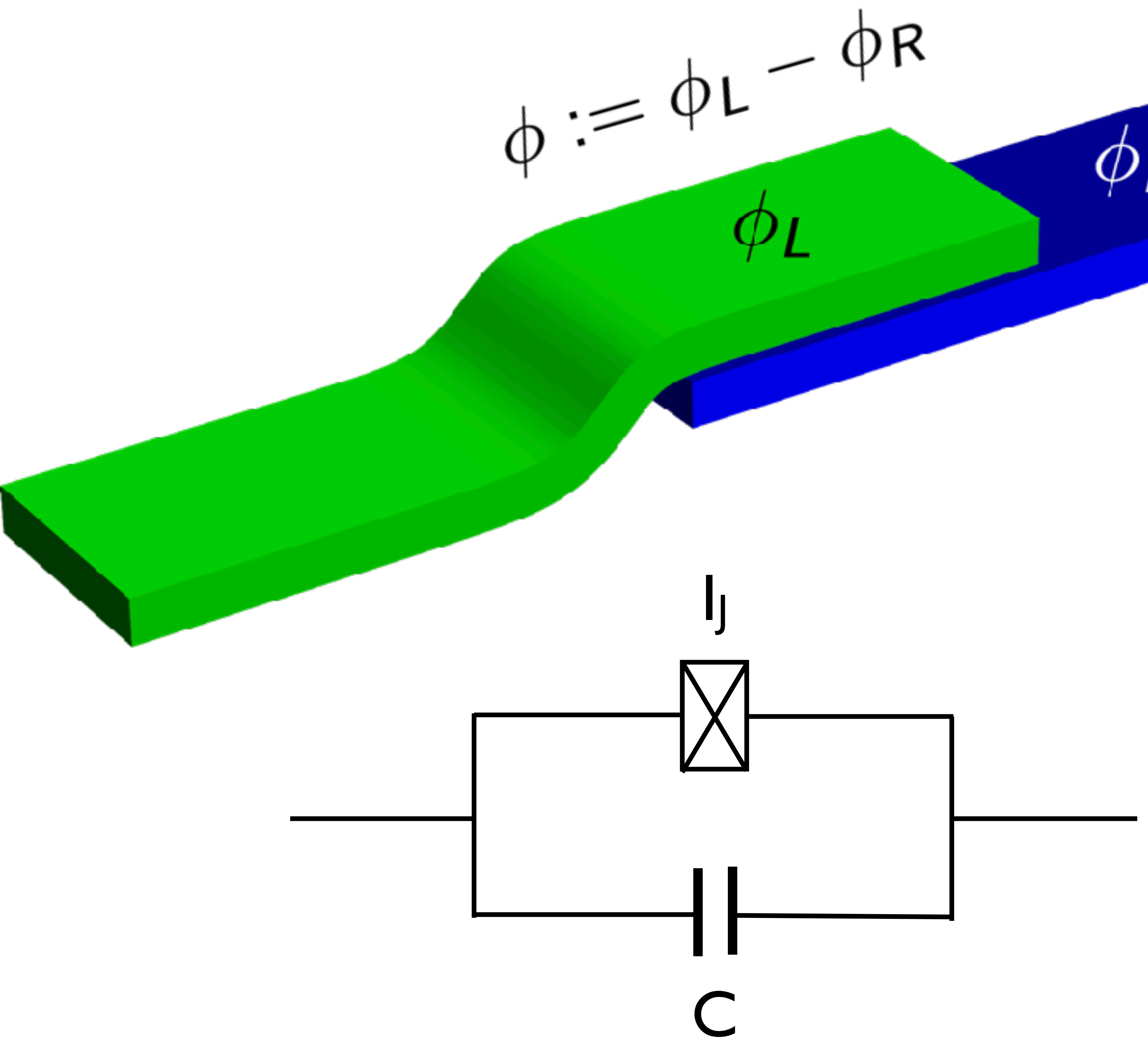
- DC Josephson Effect

$$I_S = \dot{N}_L = -\dot{N}_R = I_J \sin(\phi)$$

- AC Josephson Effect

$$\hbar\dot{\phi} = E_L - E_R = 2eV$$

Josephson Junction



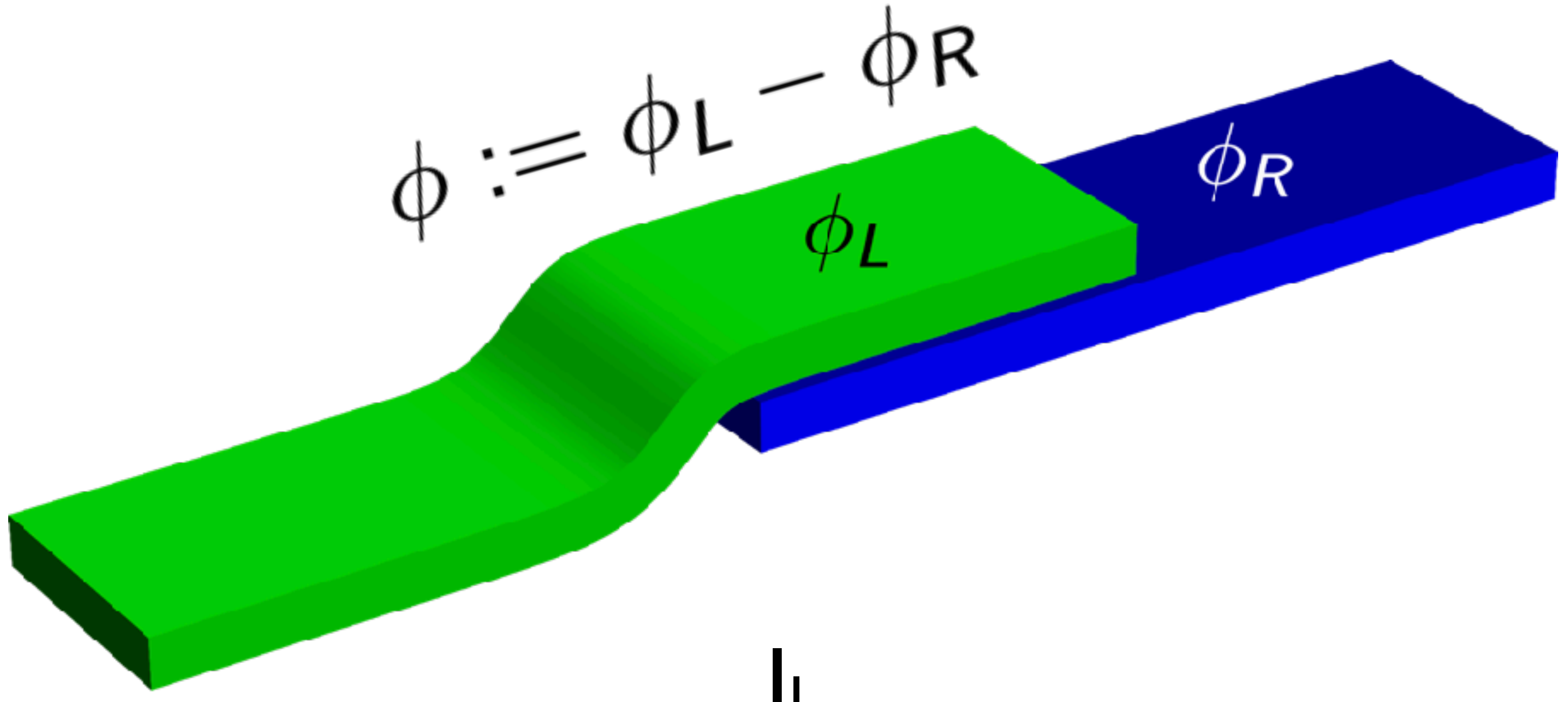
- DC Josephson Effect

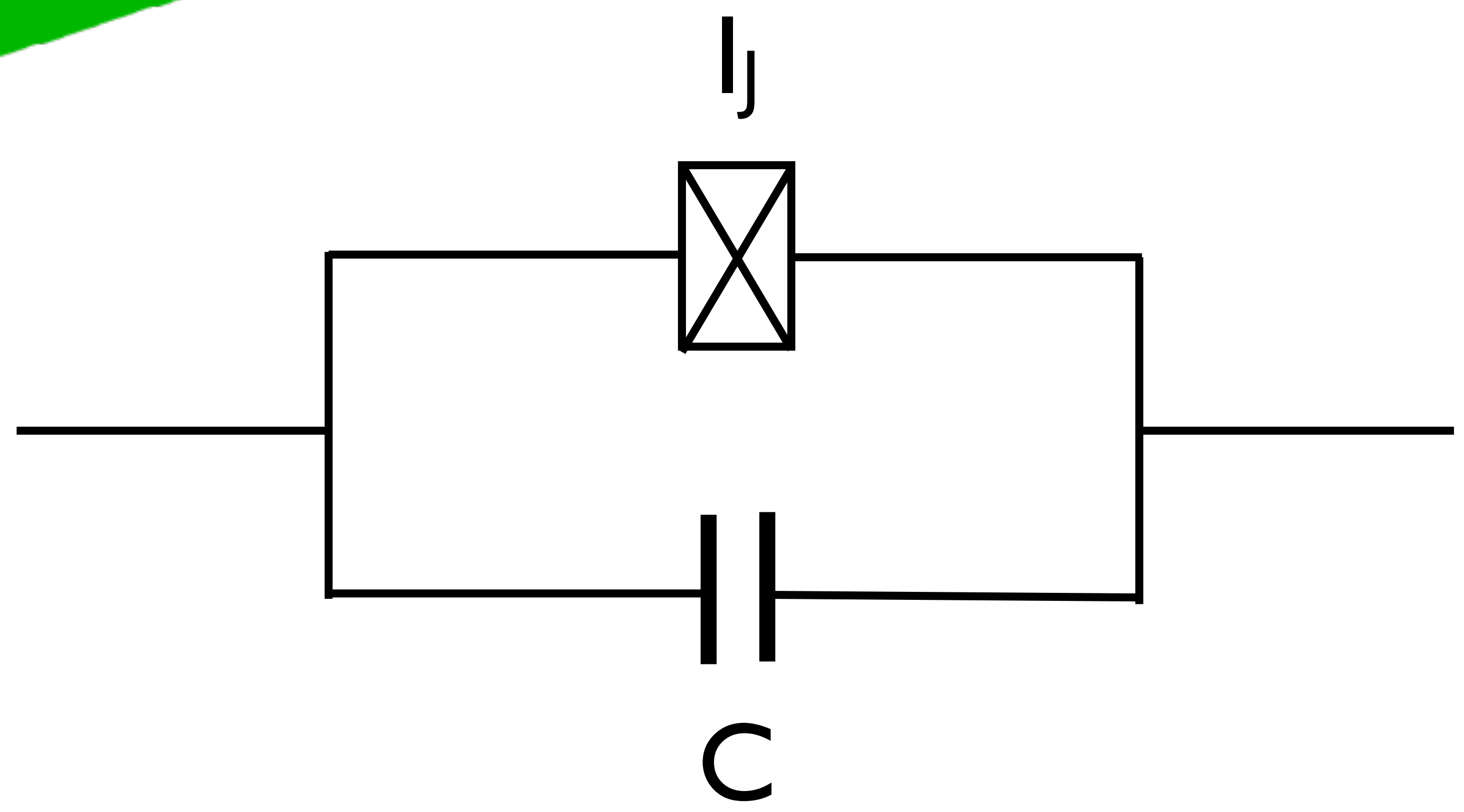
  $I_S = I_J \sin(\phi)$

- AC Josephson Effect

  $\dot{\phi} = 2eV/\hbar$

SMALL JOSEPHSON JUNCTION





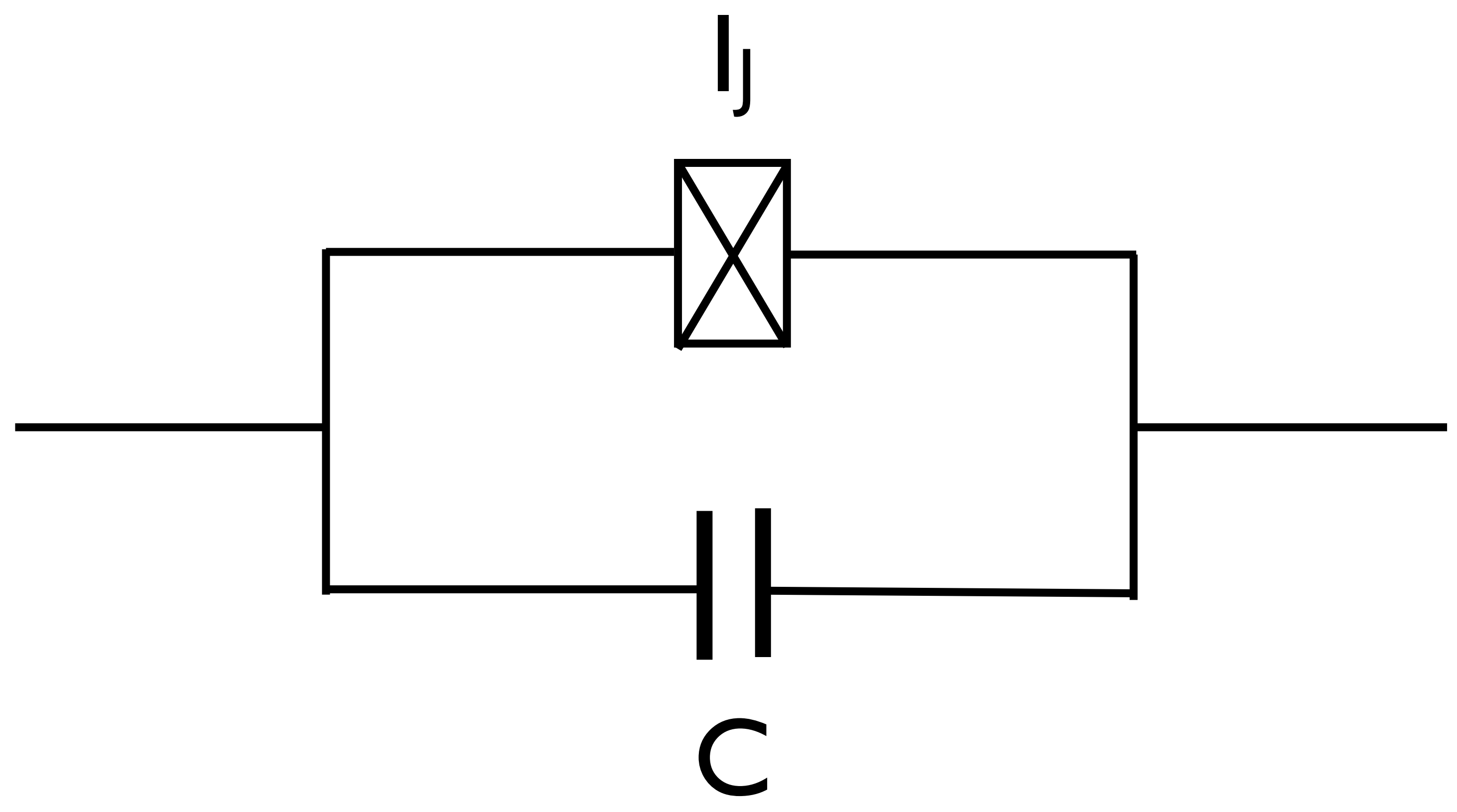
$$\dot{\phi} = \frac{2eV}{\hbar} = \frac{2e}{\hbar}\frac{Q}{C}$$

$$\dot{Q} = -I_J \sin(\phi)$$

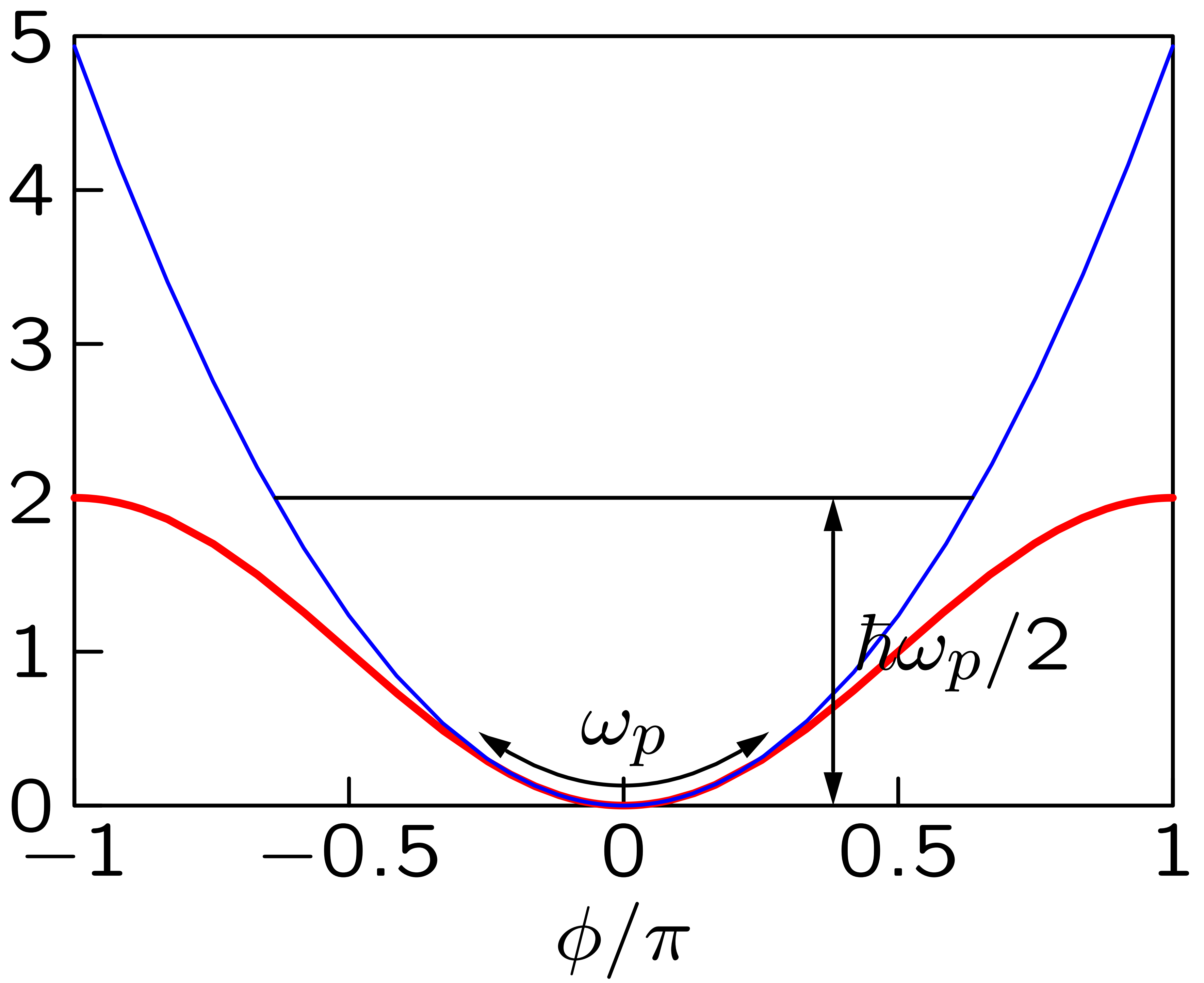
$$\ddot{\phi} = -\omega_p^2 \sin(\phi)$$

$$\omega_p^2 := \frac{2e\, I_J}{C}$$

Macroscopic Quantization



$$\hat{H} = E_C \hat{n}^2 - E_J \cos\hat{\phi}, \quad [\hat{\phi}, \hat{n}] = i$$



$$E_C := \frac{(2e)^2}{2C}$$

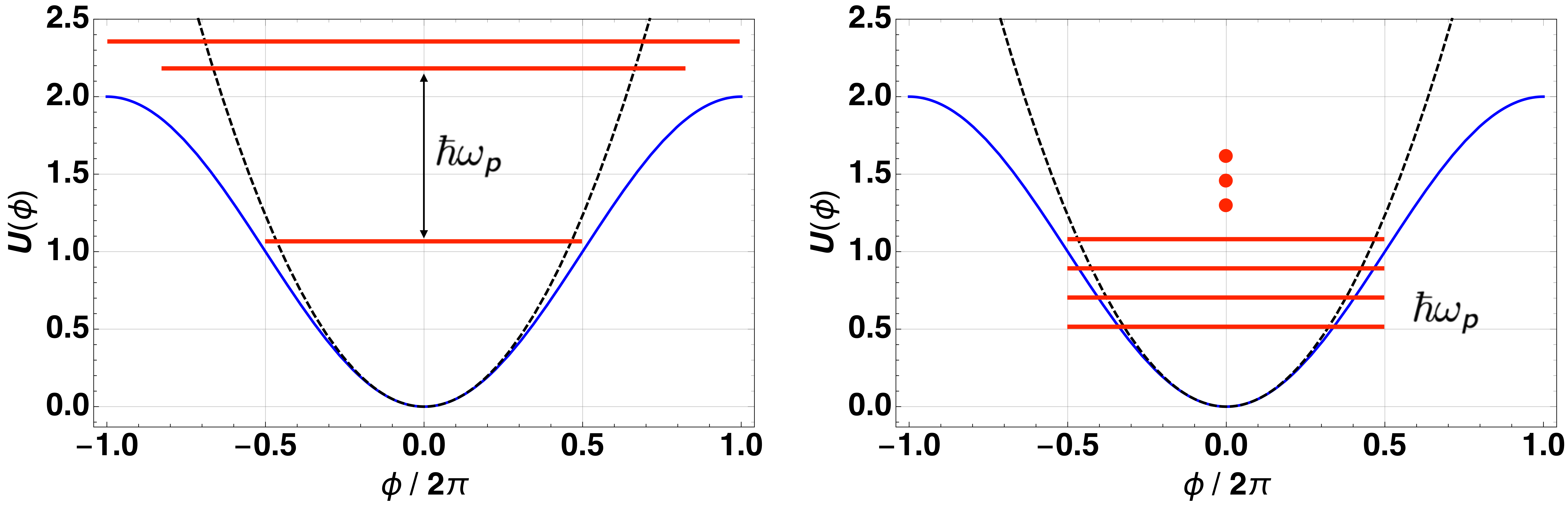
$$E_J := \frac{\hbar I_J}{2e} = \frac{I_J \Phi_0}{2\pi}$$

$$\hbar\omega_p := \sqrt{2 E_C E_J}$$

Two Extremes

$$\hat{H} = E_C \hat{n}^2 - E_J \cos\hat{\phi}, \quad [\hat{\phi}, \hat{n}] = i$$

$E_C \gg E_J$ $\qquad\qquad$ $E_C \ll E_J$

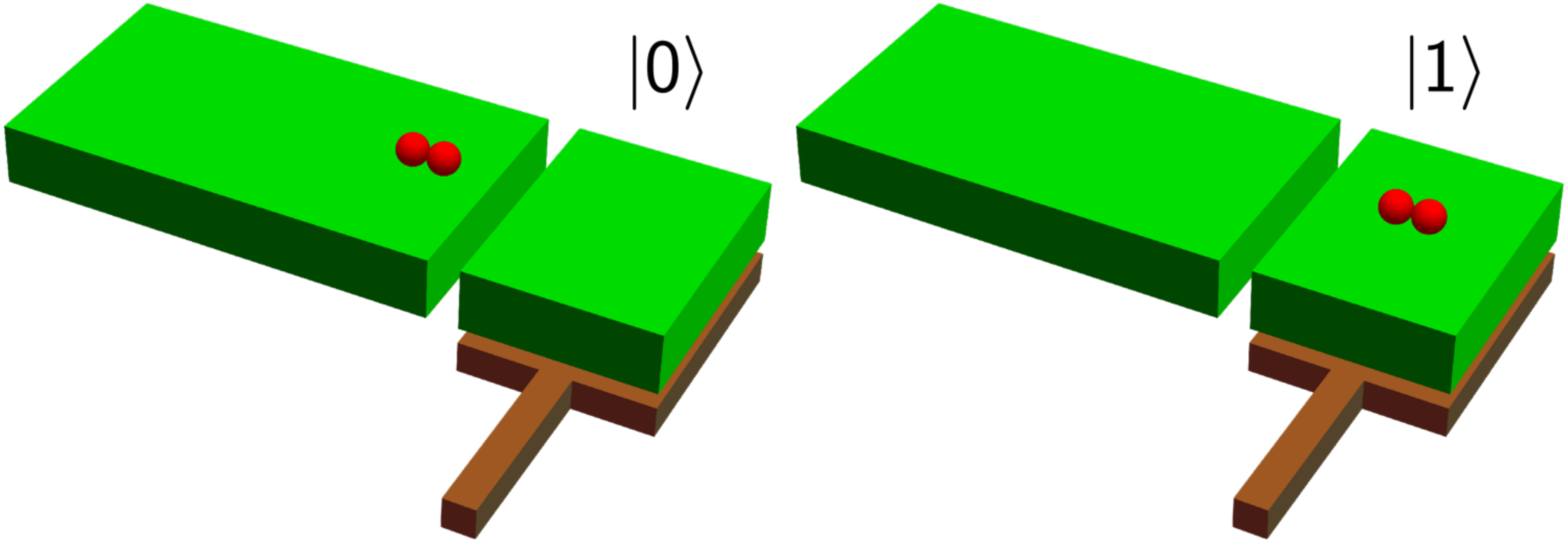


Superconducting Qubits

Charge, Phase & Transmon Qubits

Quantized Charge

(Quantized Number of Cooper Pairs)

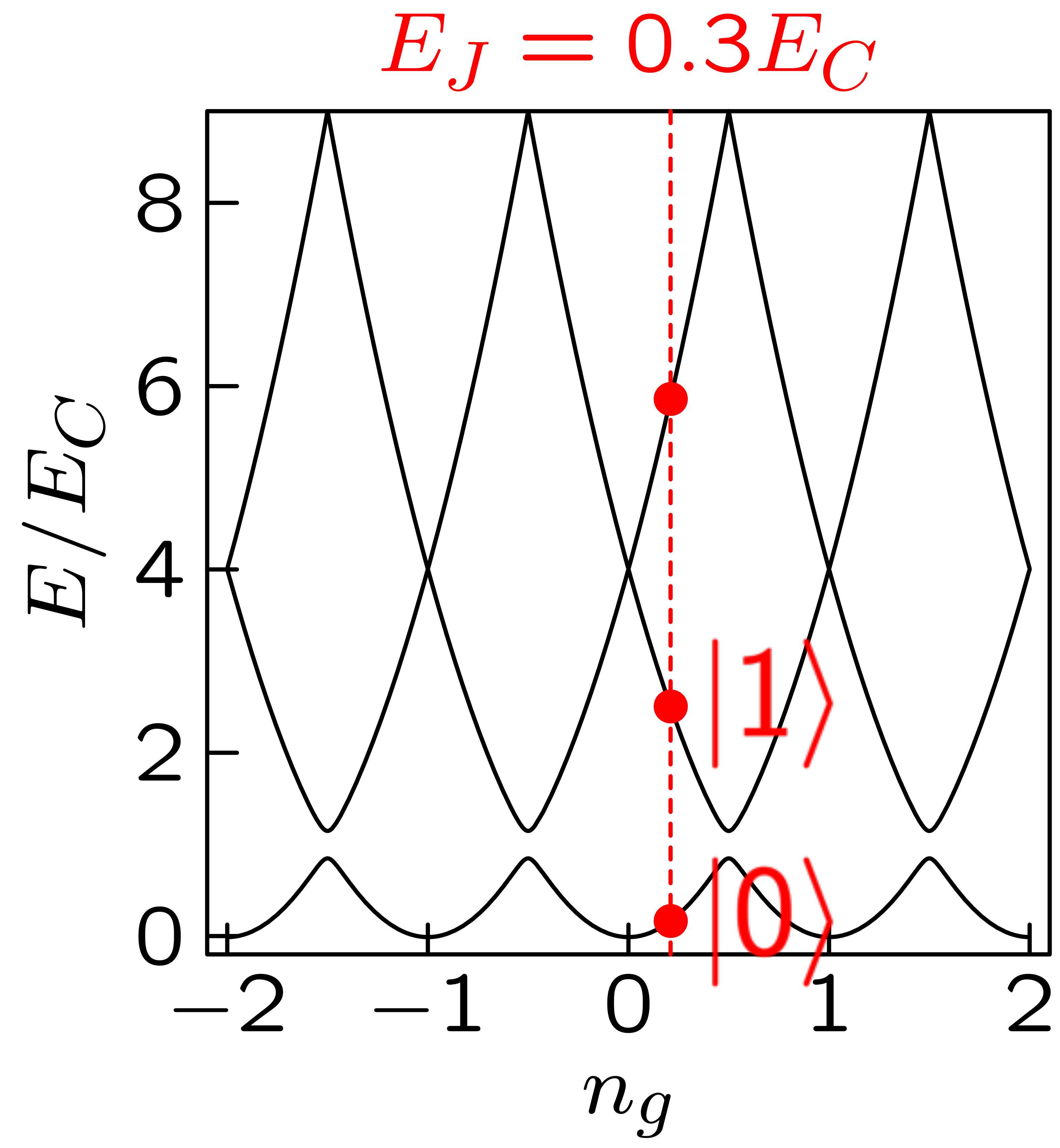


$$H_{\text{qubit}} = \frac{1}{2}\Omega\sigma^z$$

$$\Omega/2\pi \sim 5\,\text{GHz}$$

Bouchiat *et al.* (Phys. Scr., 1998)
Nakamura *et al.* (Nature, 1999)

Quantized Charge



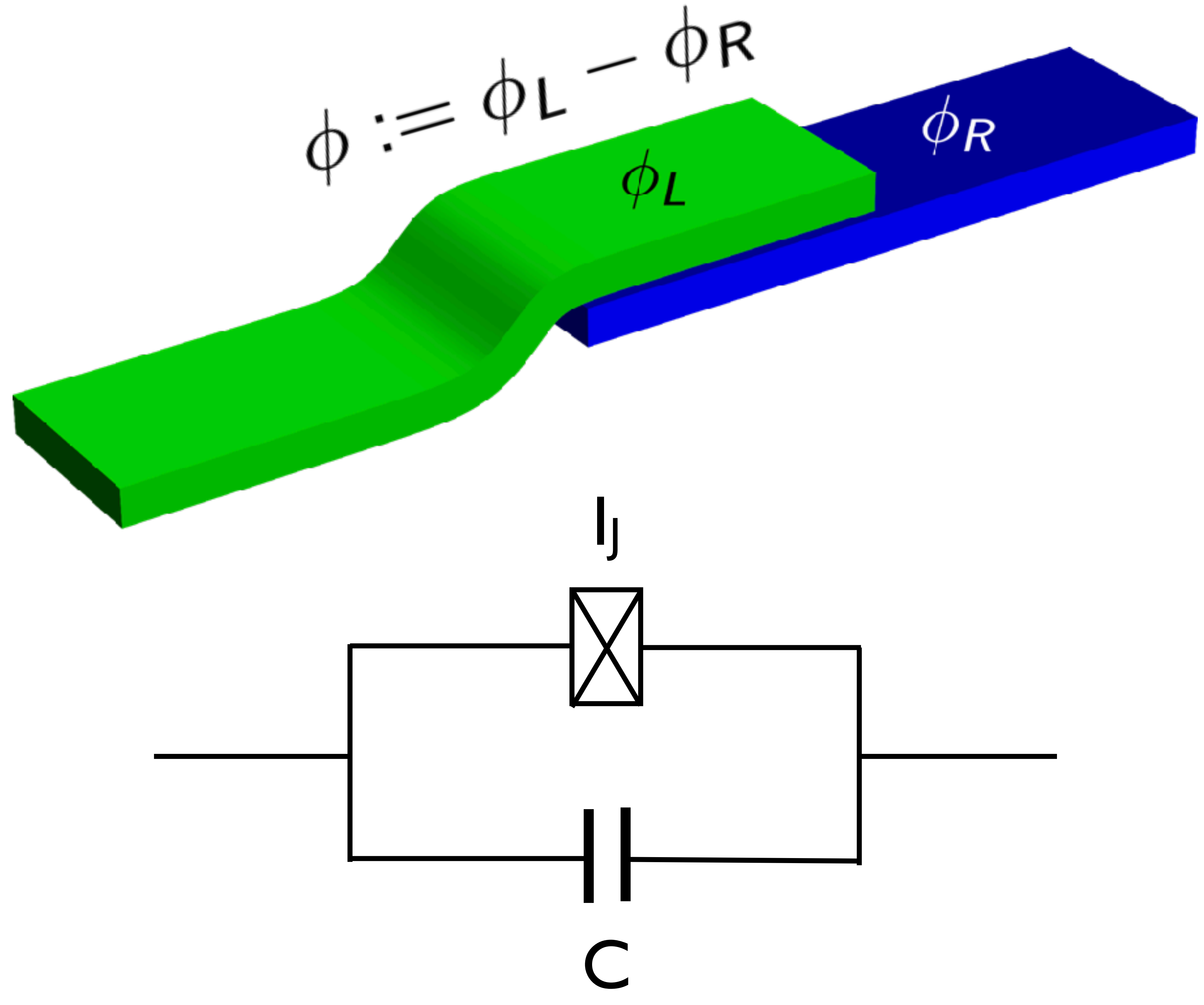
$$[\phi, n] = i$$

$$H = \overbrace{E_C(n - n_g)^2}^{\text{dominant}} - E_J \cos\phi$$

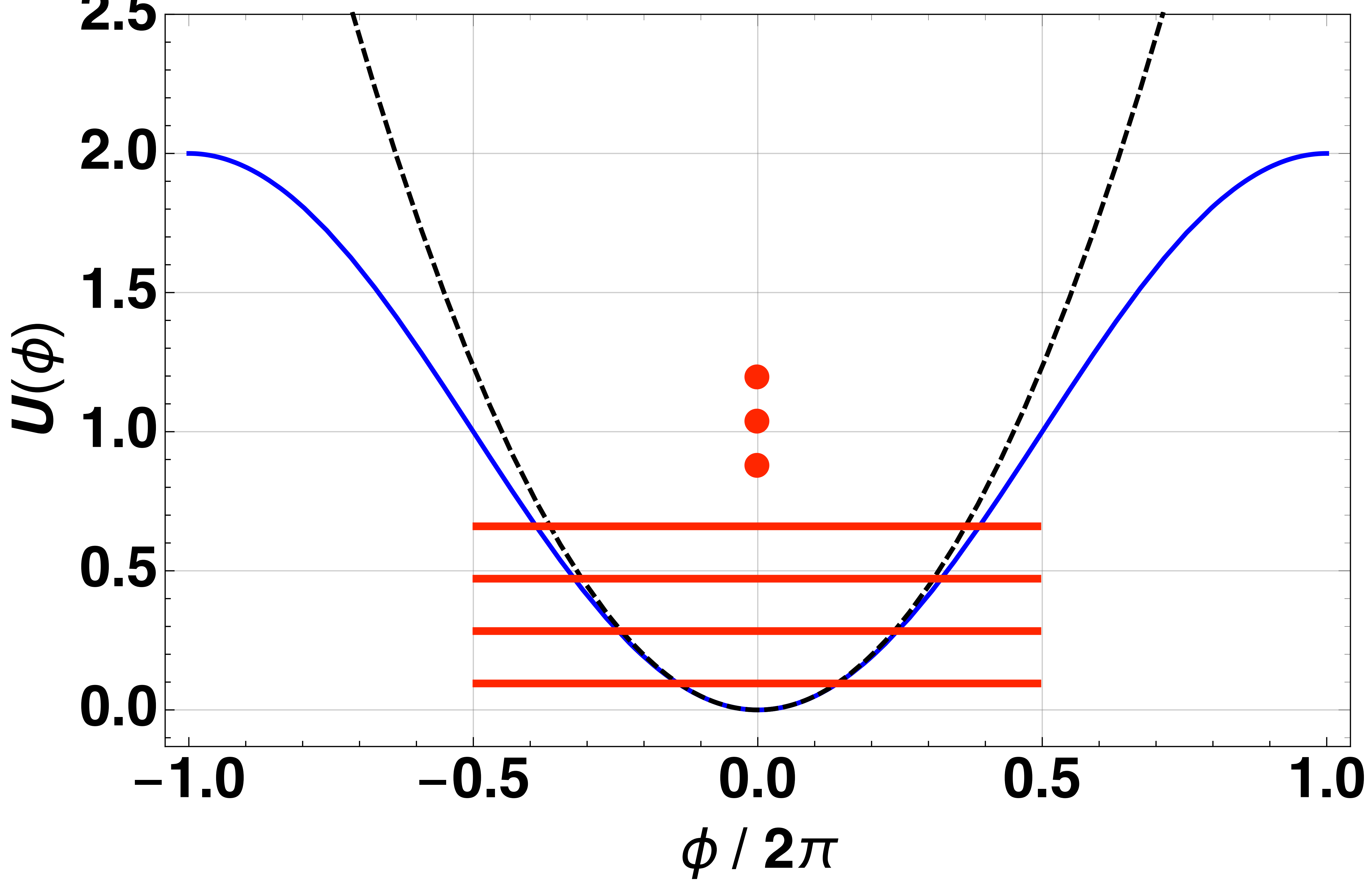
$$H \approx E_C(n_g - 1)\sigma^z - \frac{1}{2}E_J\sigma^x$$

Phase Qubit

Biased Anharmonic Oscillator

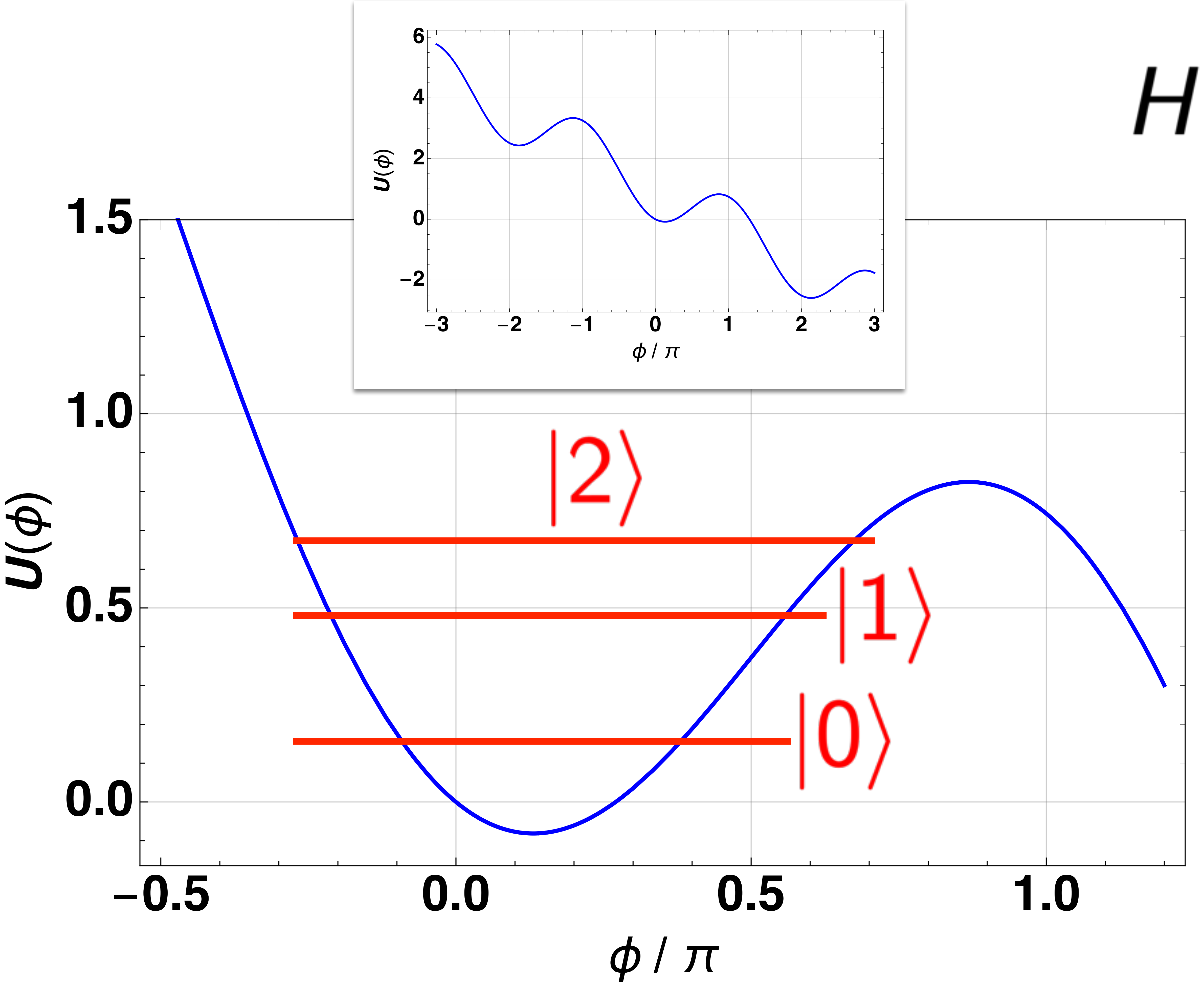


$$\frac{E_J}{E_C} \sim 10^4$$



Phase Qubit

Biased Anharmonic Oscillator

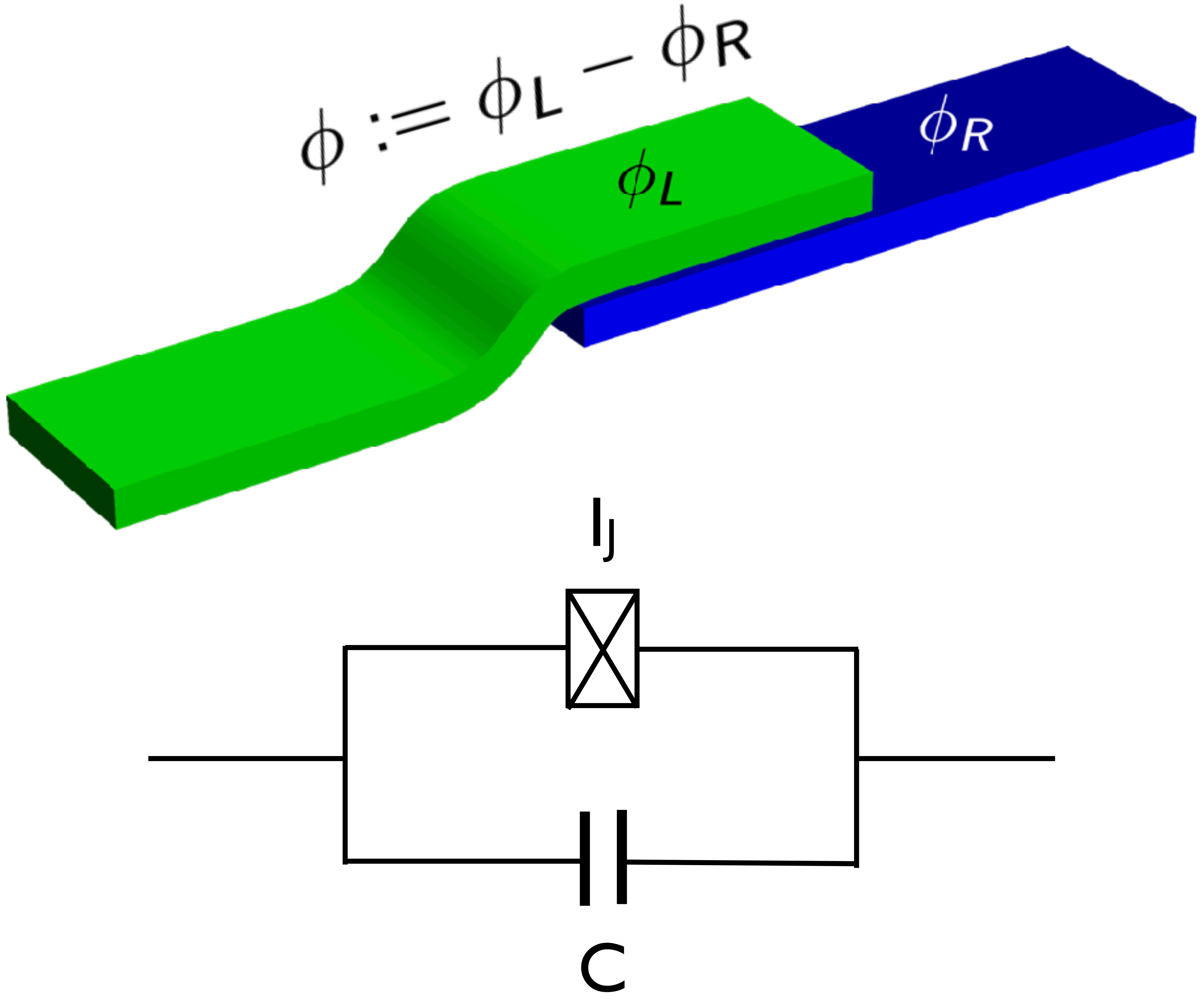


$$H = \underbrace{E_C(n - n_g)^2}_{\text{perturbation}} - E_J \cos\phi - I\phi$$

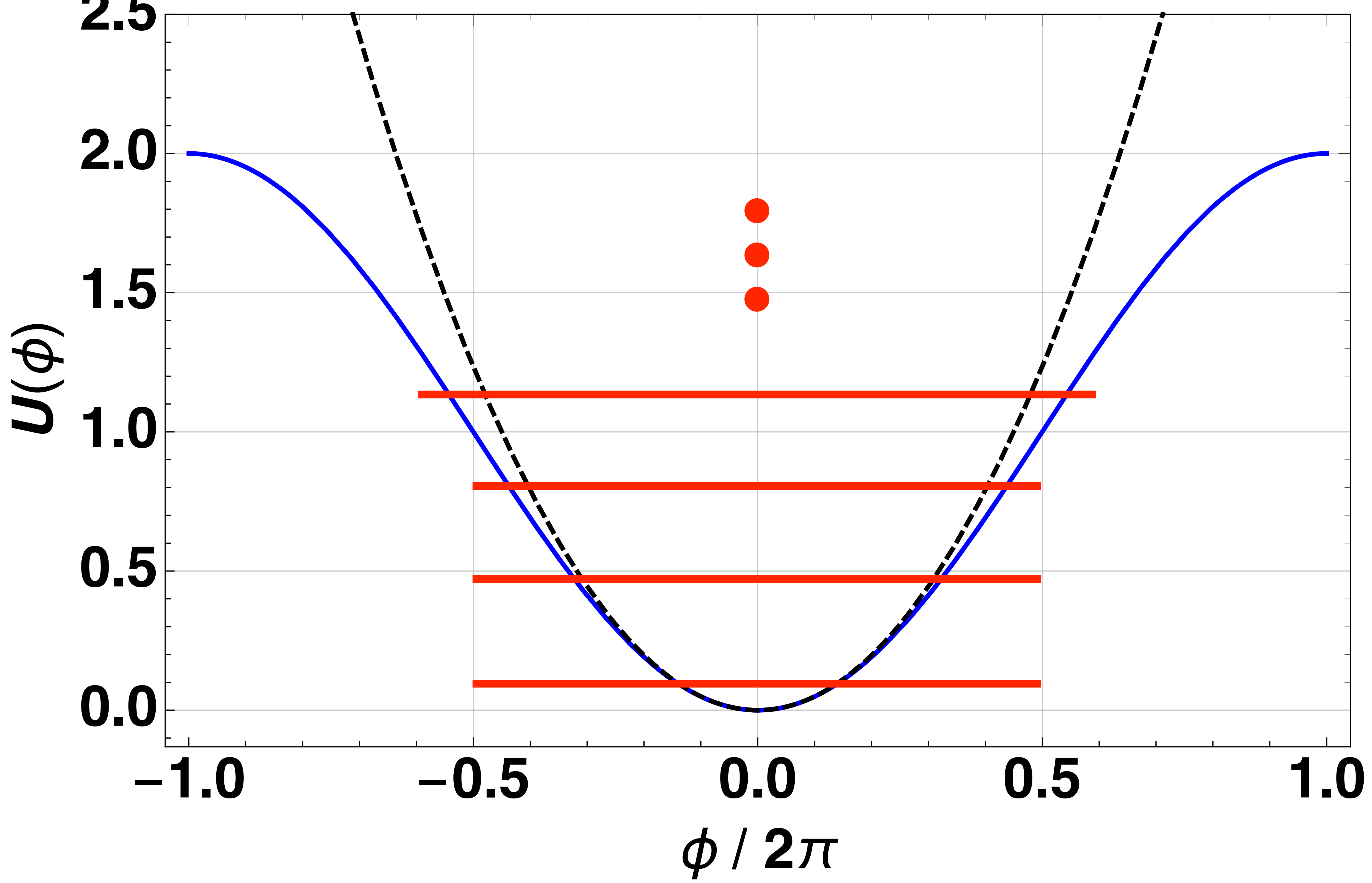
Martinis *et al.* (PRL, 2002)
Steffen *et al.* (PRL, 2006)

Quantronium / Transmon Qubit

(Unbiased) Anharmonic Oscillator



$$\frac{E_J}{E_C} \simeq 1 - 100$$

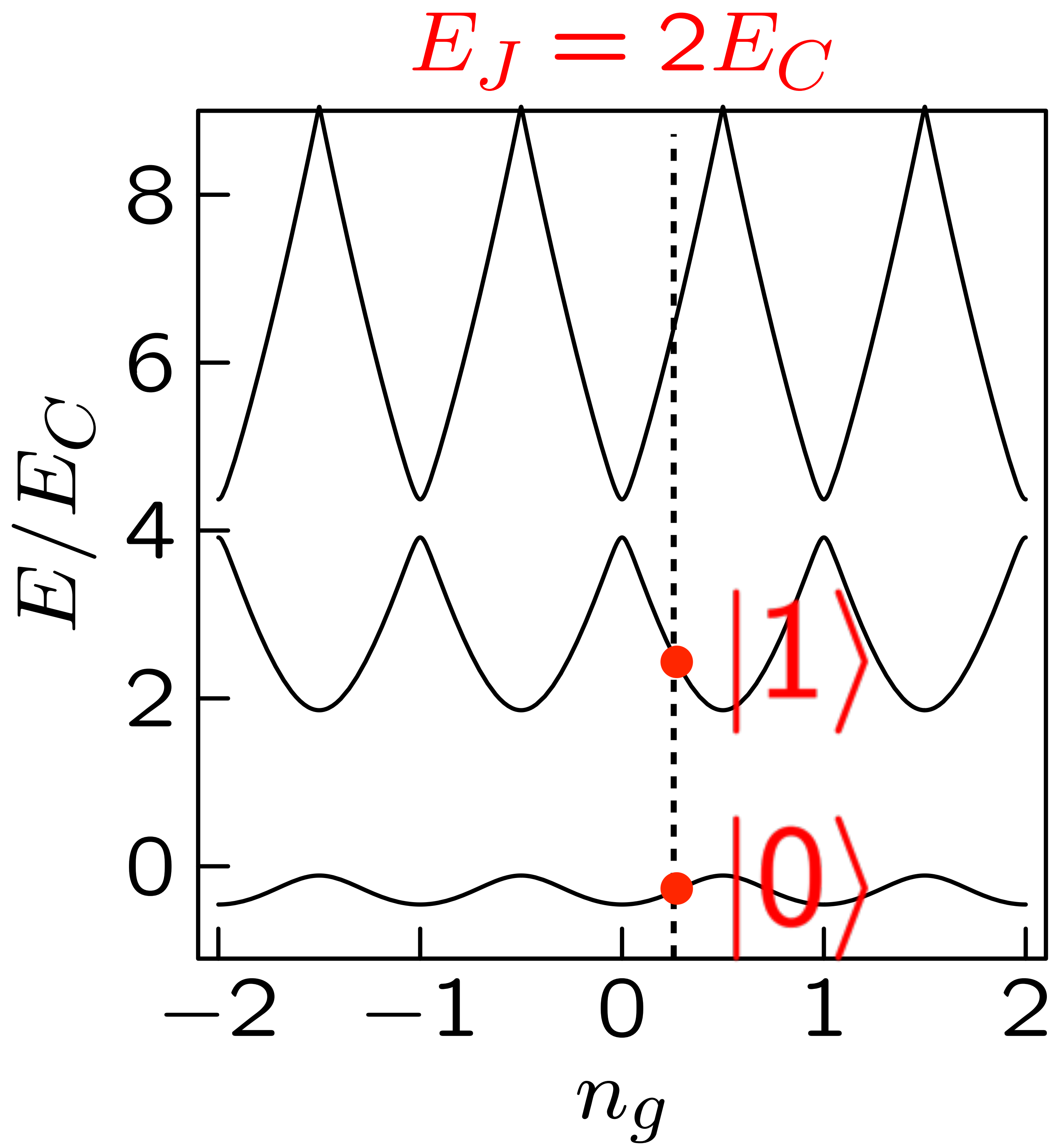


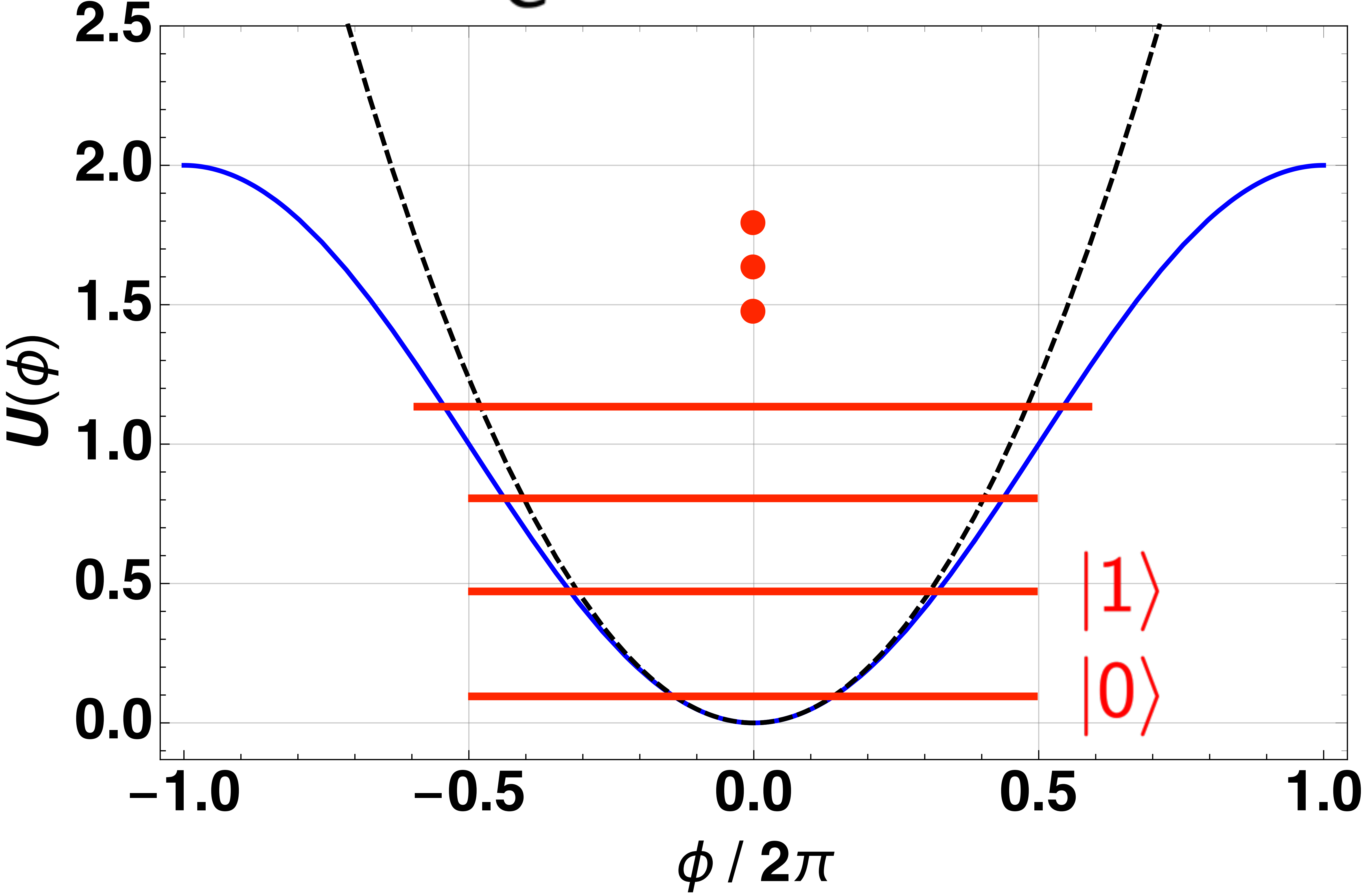
Quantronium / Transmon Qubit

(Unbiased) Anharmonic Oscillator

Cottet (PhD, 2002)
Vion *et al.* (Science, 2002)

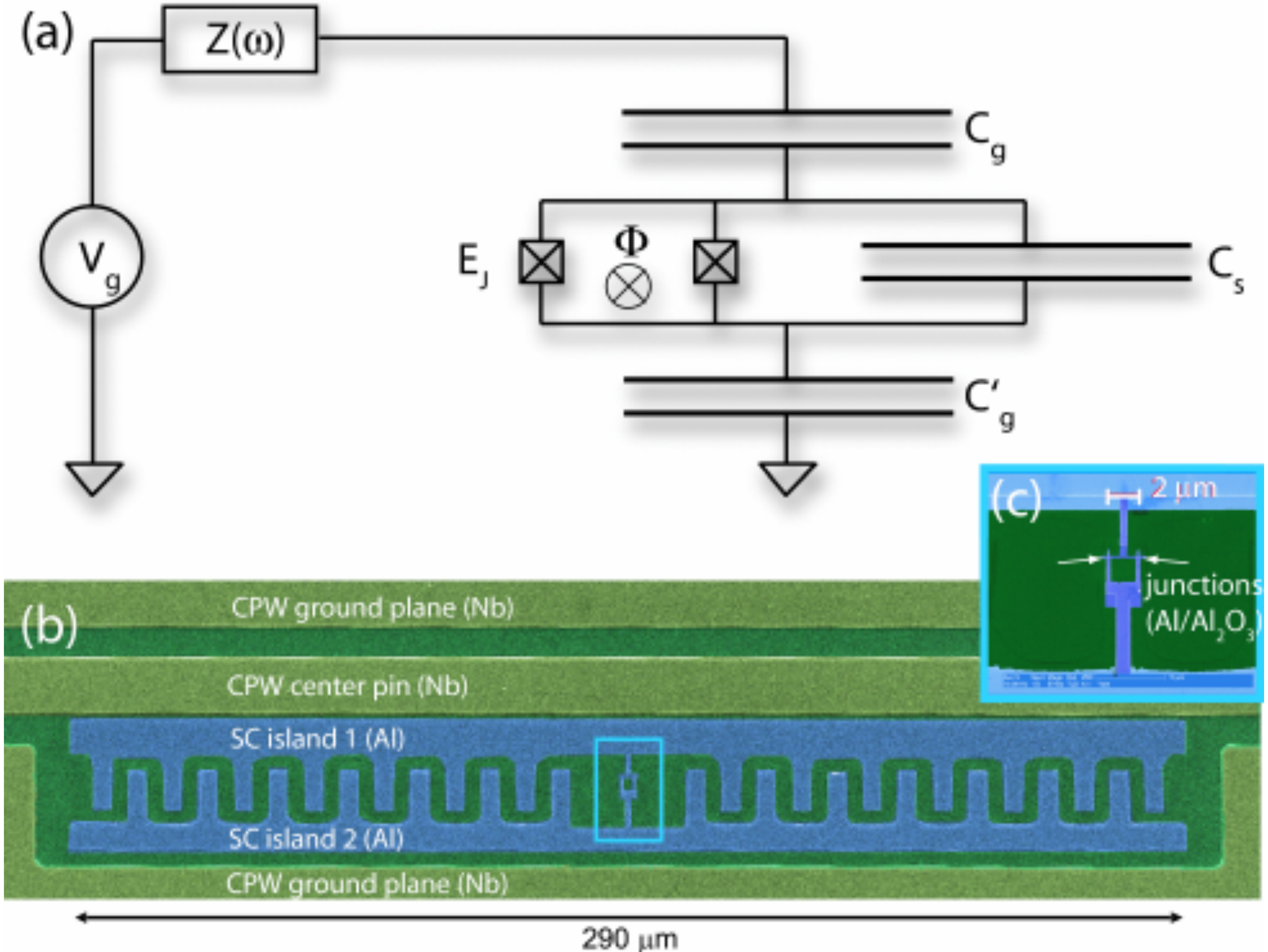
$$\frac{E_J}{E_C} \simeq 1 - 100$$





Transmon Qubit (Atom)

Koch et al. (PRA, 2007)
Houck et al. (Nature, 2007)



$$\Omega/2\pi \sim 5\,\mathrm{GHz}$$

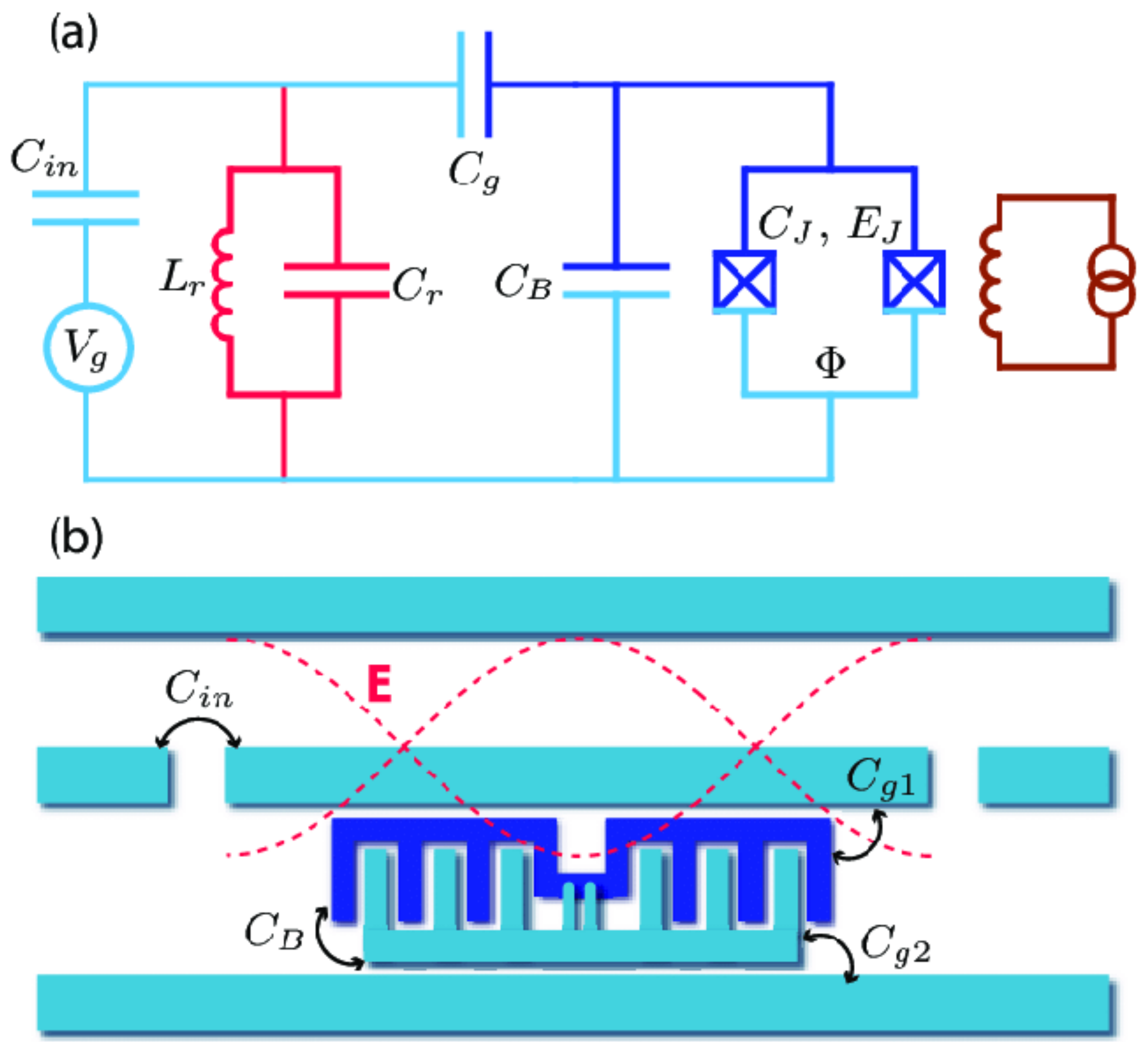
$$H_{\mathrm{qubit}} = \frac{1}{2}\Omega\sigma^z$$

Transmon Qubit

Anharmonic Oscillator

$$\frac{E_J}{E_C} \simeq 100$$

Houck et al. (Nature, 2007)
Koch *et al.* (PRA, 2007)



$$H_{\text{qubit}} = \frac{1}{2}\Omega\sigma^z$$

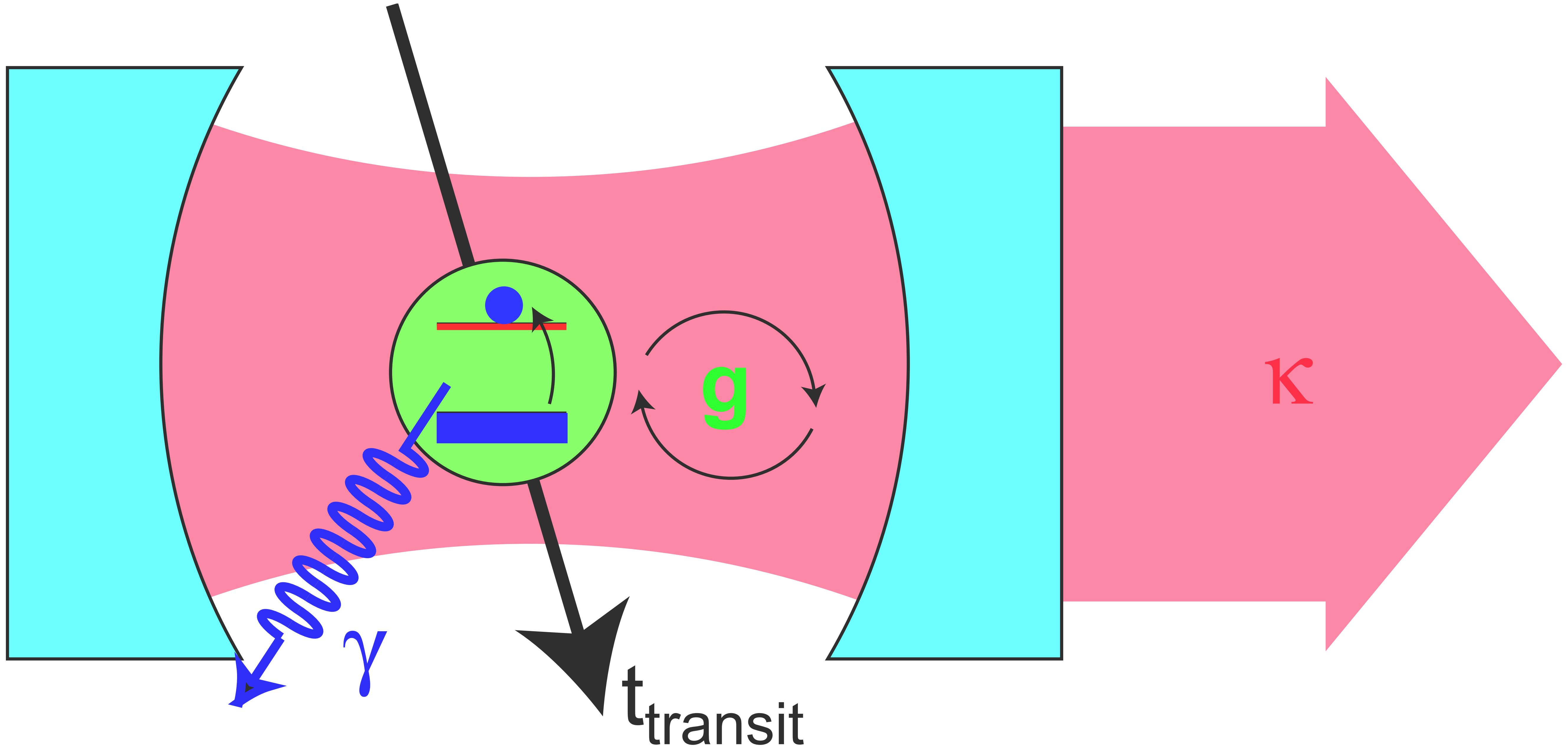
$$\Omega/2\pi \sim 5\,\text{GHz}$$

Circuit QED

(Cavity QED system based on superconducting circuit)

"Traditional" Cavity QED System

Fundamental Light-Matter Interaction



ω : photon frequency

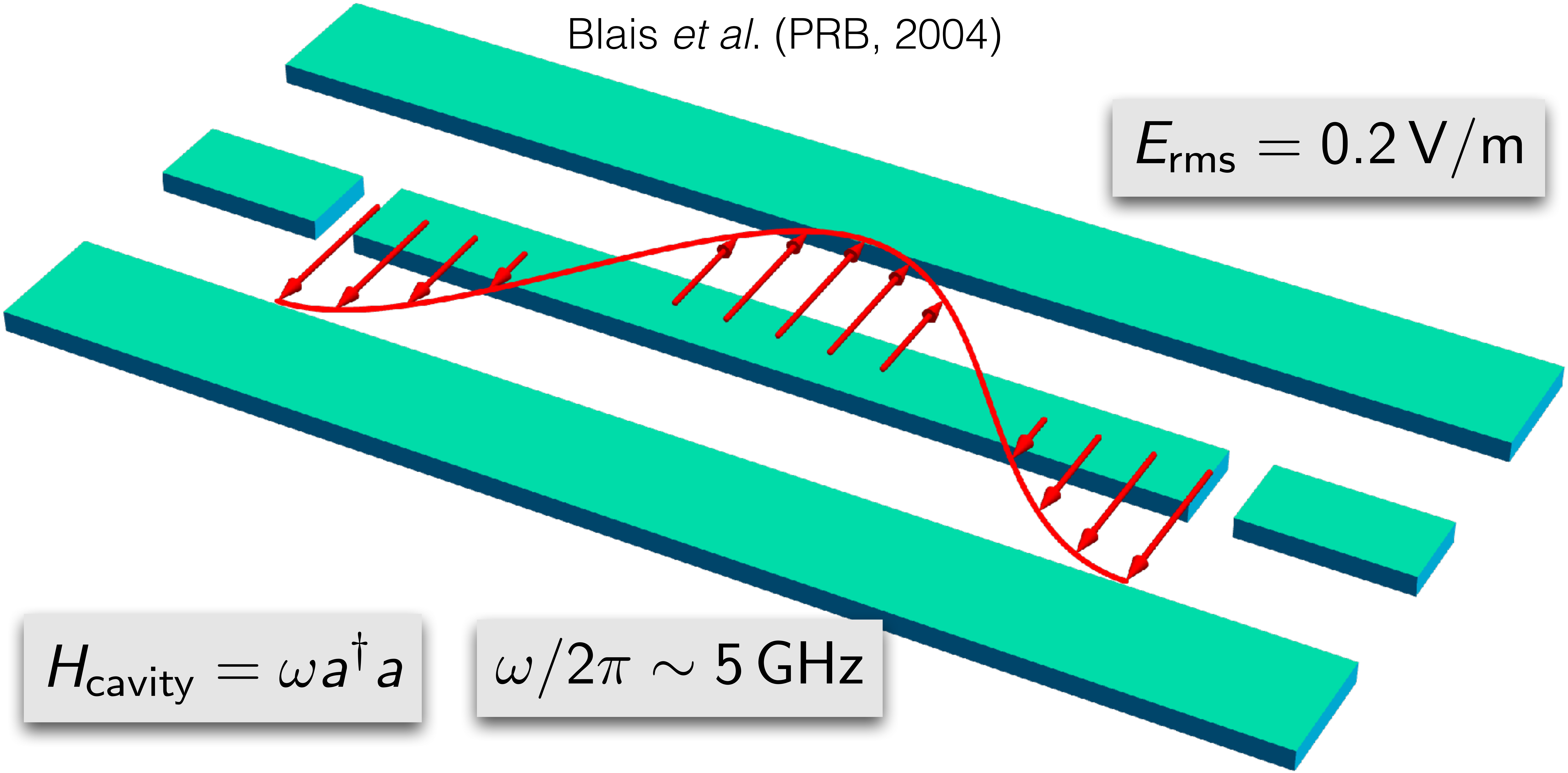
Ω : atomic transition frequency

κ : photon loss rate

γ : atomic decoherence rate

Resonator (Cavity)

Blais *et al.* (PRB, 2004)



$$E_{\mathrm{rms}} = 0.2\,\mathrm{V/m}$$

$$H_{\mathrm{cavity}} = \omega a^\dagger a$$

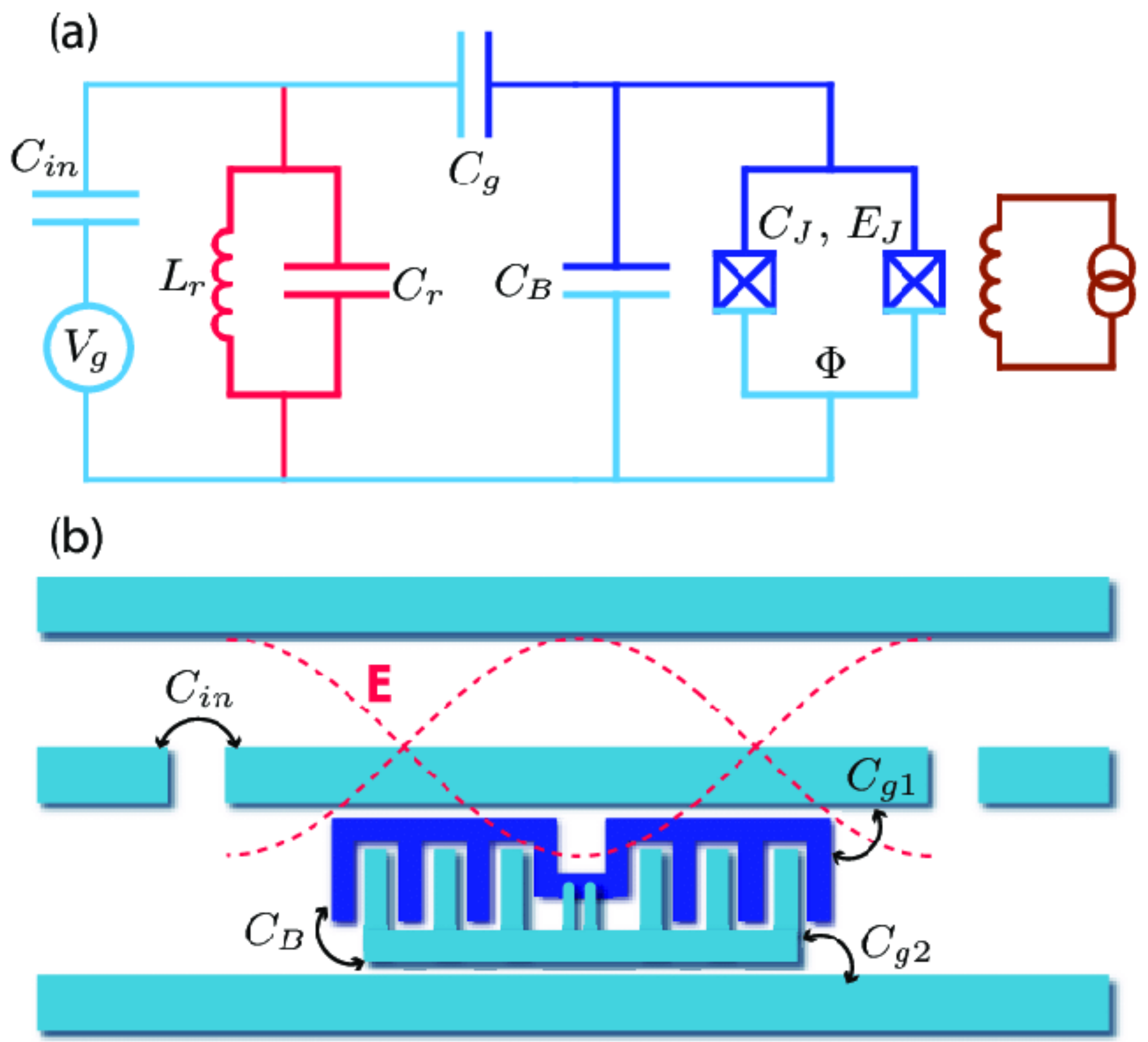
$$\omega/2\pi \sim 5\,\mathrm{GHz}$$

Transmon Qubit

Anharmonic Oscillator

$$\frac{E_J}{E_C} \simeq 100$$

Houck et al. (Nature, 2007)
Koch *et al.* (PRA, 2007)

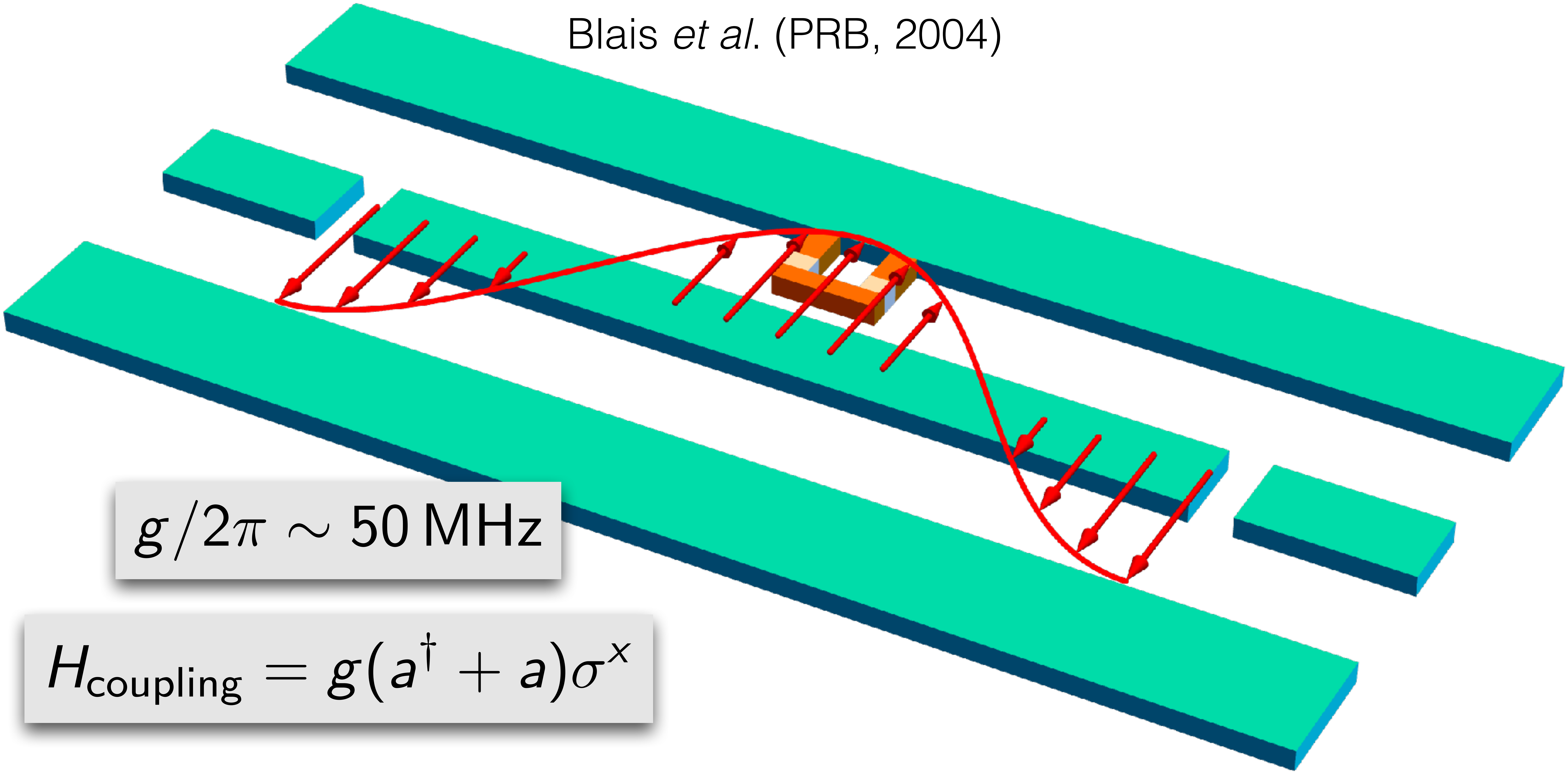


$$H_{\text{qubit}} = \frac{1}{2}\Omega\sigma^z$$

$$\Omega/2\pi \sim 5\,\text{GHz}$$

Circuit QED System

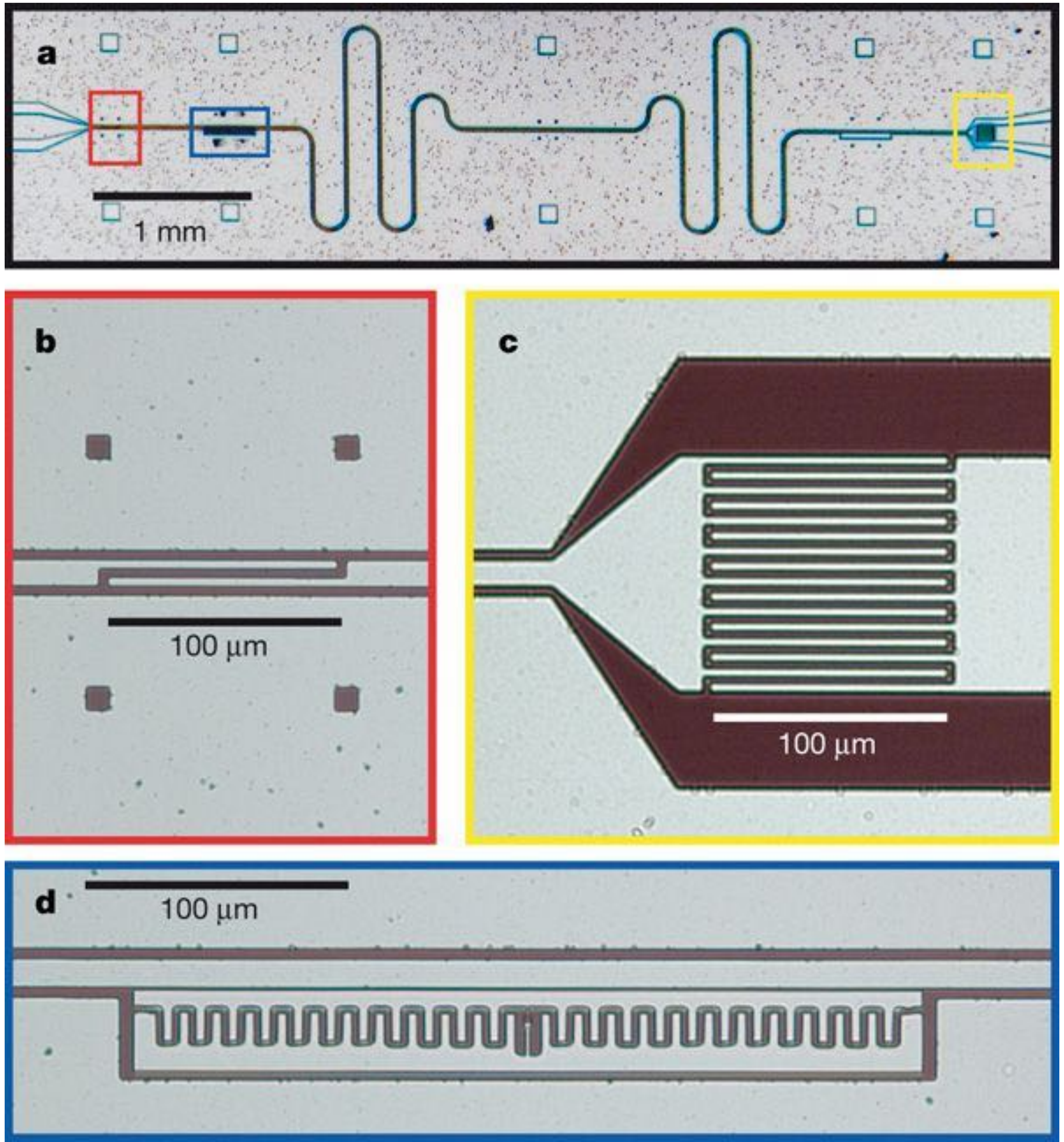
Blais *et al*. (PRB, 2004)



$$g/2\pi \sim 50\,\mathrm{MHz}$$

$$H_{\text{coupling}} = g(a^\dagger + a)\sigma^x$$

Circuit QED System



$$g/2\pi \approx 100\,\mathrm{MHz}$$

$$H_{\mathrm{coupling}} = g(a^\dagger + a)\sigma^x$$

Houck *et al.* (Nature, 2007)

The Rabi Model

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

- κ : cavity photon loss rate
- γ : qubit decoherence rate

$$\omega/2\pi \sim \Omega/2\pi \sim 5\,\mathrm{GHz}$$

$$g/2\pi \sim 50\,\mathrm{MHz}$$

$$\boxed{g/\omega \sim 10^{-3}\text{–}1}$$

- Bishop *et al*., Nature Physics (2009)
- Niemczyk *et al*., Nature Physics (2010)
- …

So, let's solve the Rabi

The Rabi Model

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + \boxed{g(a^\dagger + a)\sigma^x}$$

- κ : cavity photon loss rate
- γ : qubit decoherence rate

Strong Coupling, Ultra Strong Coupling, Resonance, ...

$$g \gg \kappa, \gamma$$

"strong" coupling

$$g \sim \sqrt{\omega\Omega}$$

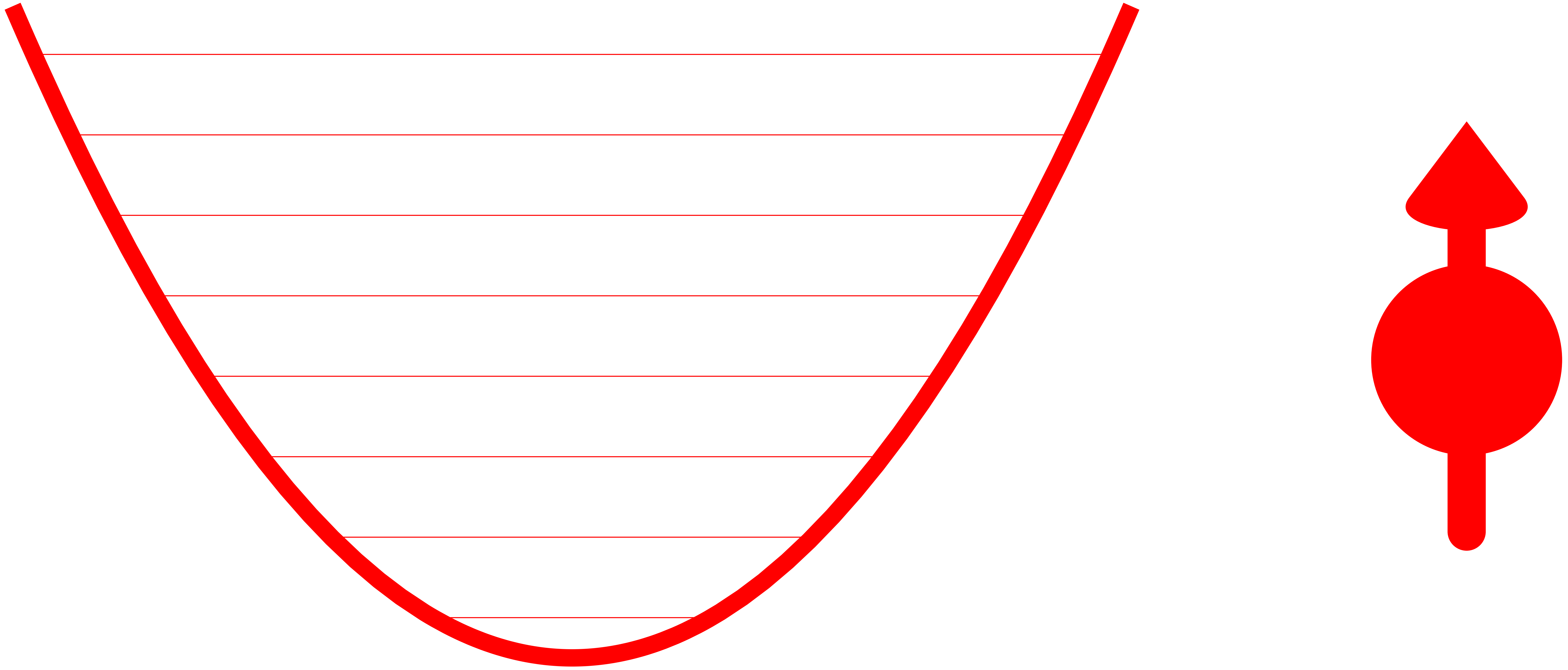
"ultra strong" coupling

$$\omega = \Omega$$

resonance

Weak-Coupling Limit

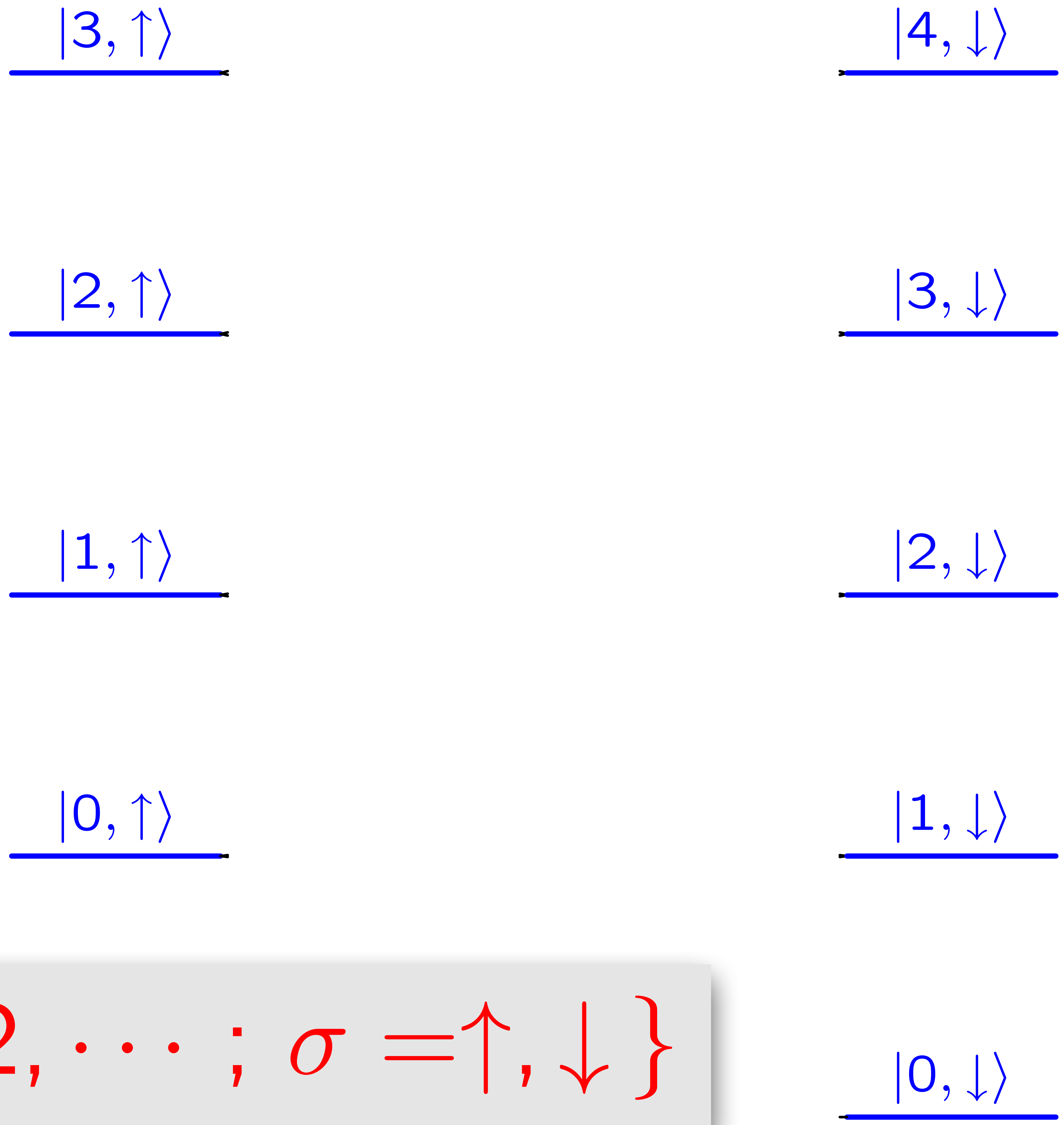
$$g = 0$$



$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + \cancel{g(a^\dagger + a)\sigma^x}$$

Weak-Coupling Limit

$g = 0$



$$\{\, |n,\sigma\rangle \mid n = 0, 1, 2, \cdots ;\ \sigma = \uparrow, \downarrow \,\}$$

"STRONG" COUPLING

Jaynes-Cummings Model (Rotating Wave Approximation)

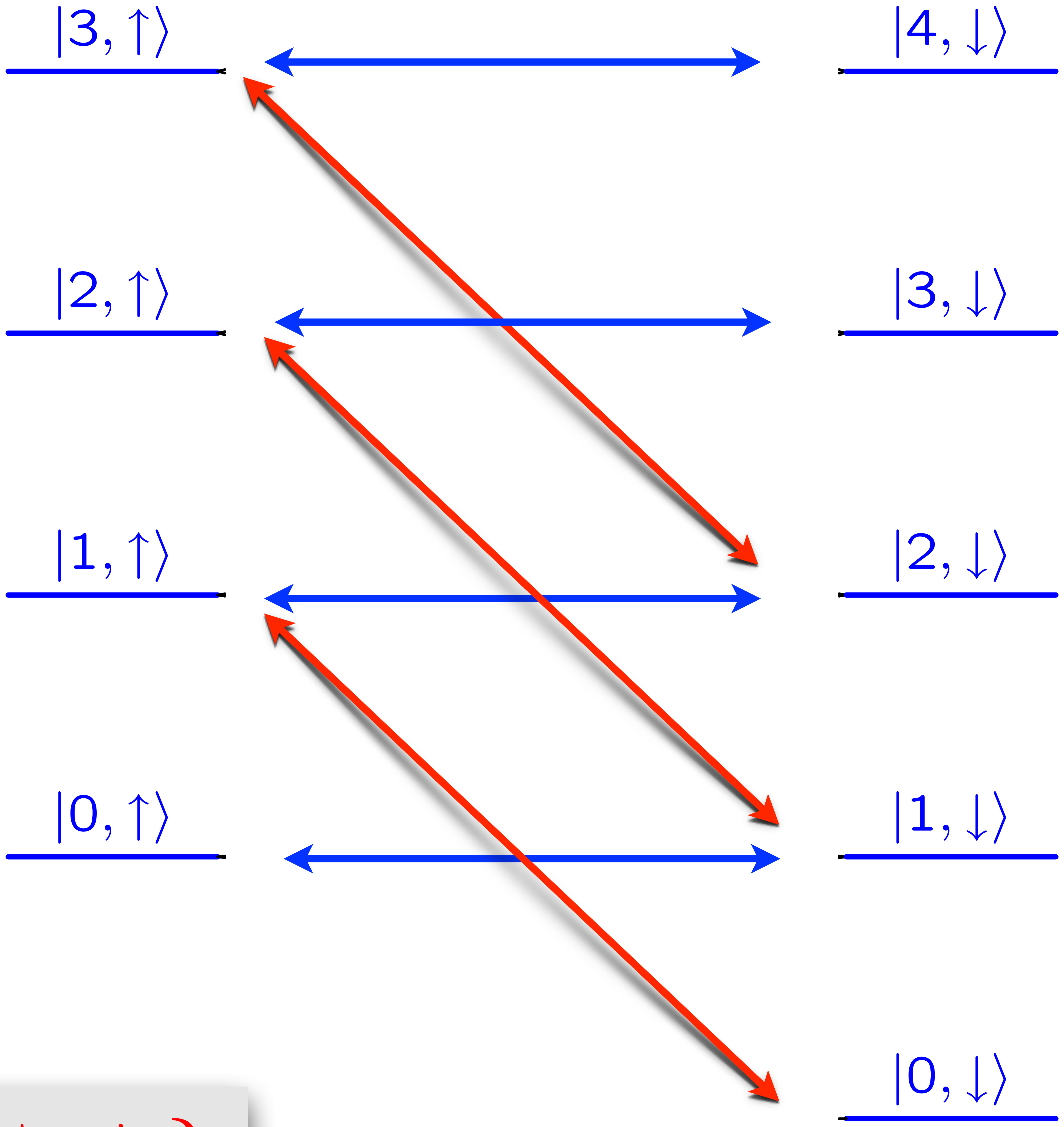
$$\boxed{\kappa, \gamma \ll g \ll \omega}$$

$$(a^\dagger + a)(\sigma^+ + \sigma^-) = \underbrace{(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \underbrace{(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}$$

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

Rotating vs. Counter-Rotating

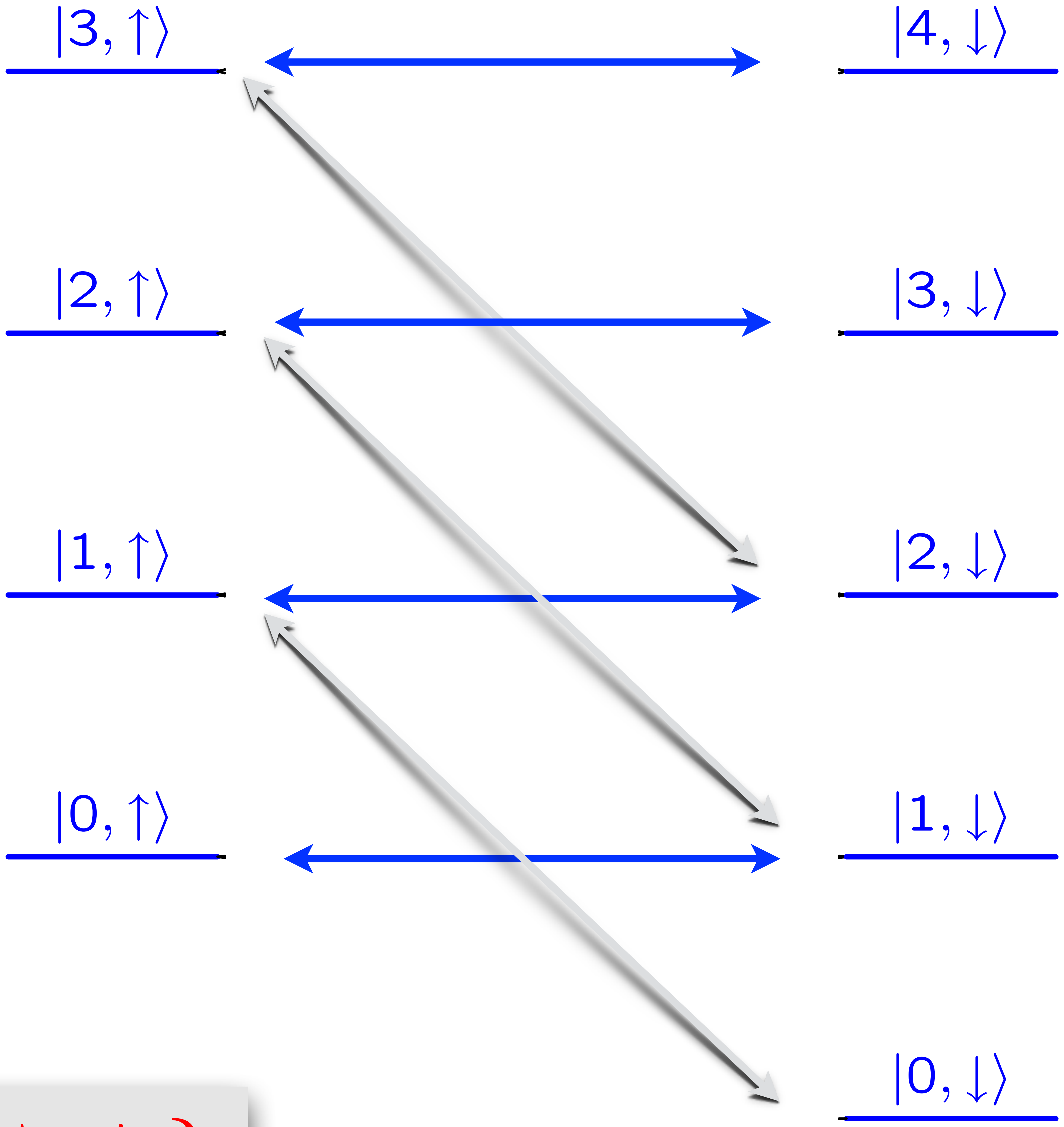
$$\underbrace{(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \underbrace{(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}$$



$$\{\,|n,\sigma\rangle \mid n = 0,1,2,\cdots;\ \sigma = \uparrow,\downarrow\,\}$$

Rotating vs. ~~Counter-Rotating~~

$$\underbrace{(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \cancel{\underbrace{(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}}$$



$$\{\, |n,\sigma\rangle \mid n = 0, 1, 2, \cdots ;\ \sigma = \uparrow, \downarrow \,\}$$

The Jaynes-Cummings Model

(Rotating Wave Approximation)

$$\boxed{H_{JC} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a\sigma^+ + a^\dagger\sigma^-)} + \underbrace{g(a\sigma^- + a^\dagger\sigma^+)}_{\text{counter-rotating}}$$

- κ : cavity photon loss rate
- γ : qubit decoherence rate

The Jaynes-Cummings Model

(Rotating Wave Approximation)

$$\boxed{H_{JC} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a\sigma^+ + a^\dagger\sigma^-)} + \underbrace{g(a\sigma^- + a^\dagger\sigma^+)}_{\text{counter-rotating}}$$

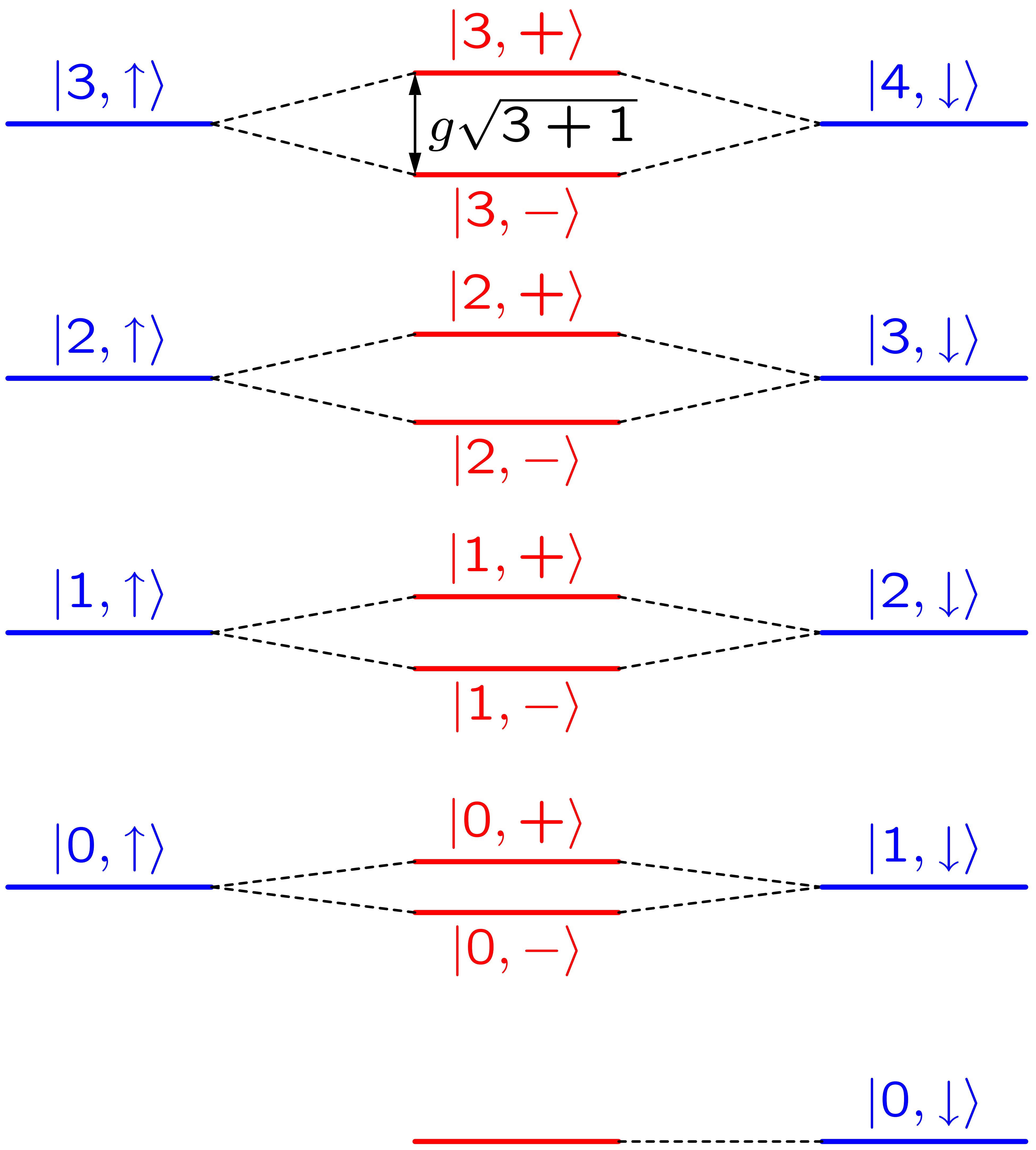
$$\begin{tikzcd} |n,\uparrow\rangle \arrow[rr, bend left] & & |n+1,\downarrow\rangle \arrow[ll, bend left] \end{tikzcd}$$

$$\begin{bmatrix} |n,\uparrow\rangle \\ |n+1,\downarrow\rangle \end{bmatrix} = \left\{ (n+1/2)\omega + \begin{bmatrix} (\Omega-\omega)/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & -(\Omega-\omega)/2 \end{bmatrix} \right\} \begin{bmatrix} |n,\uparrow\rangle \\ |n+1,\downarrow\rangle \end{bmatrix}$$

$$E_{n,\pm} = (n+1/2)\omega \pm \sqrt{g^2(n+1) + (\Omega-\omega)^2/4}$$

“STRONG” COUPLING

Jaynes-Cummings Model (Rotating Wave Approximation)

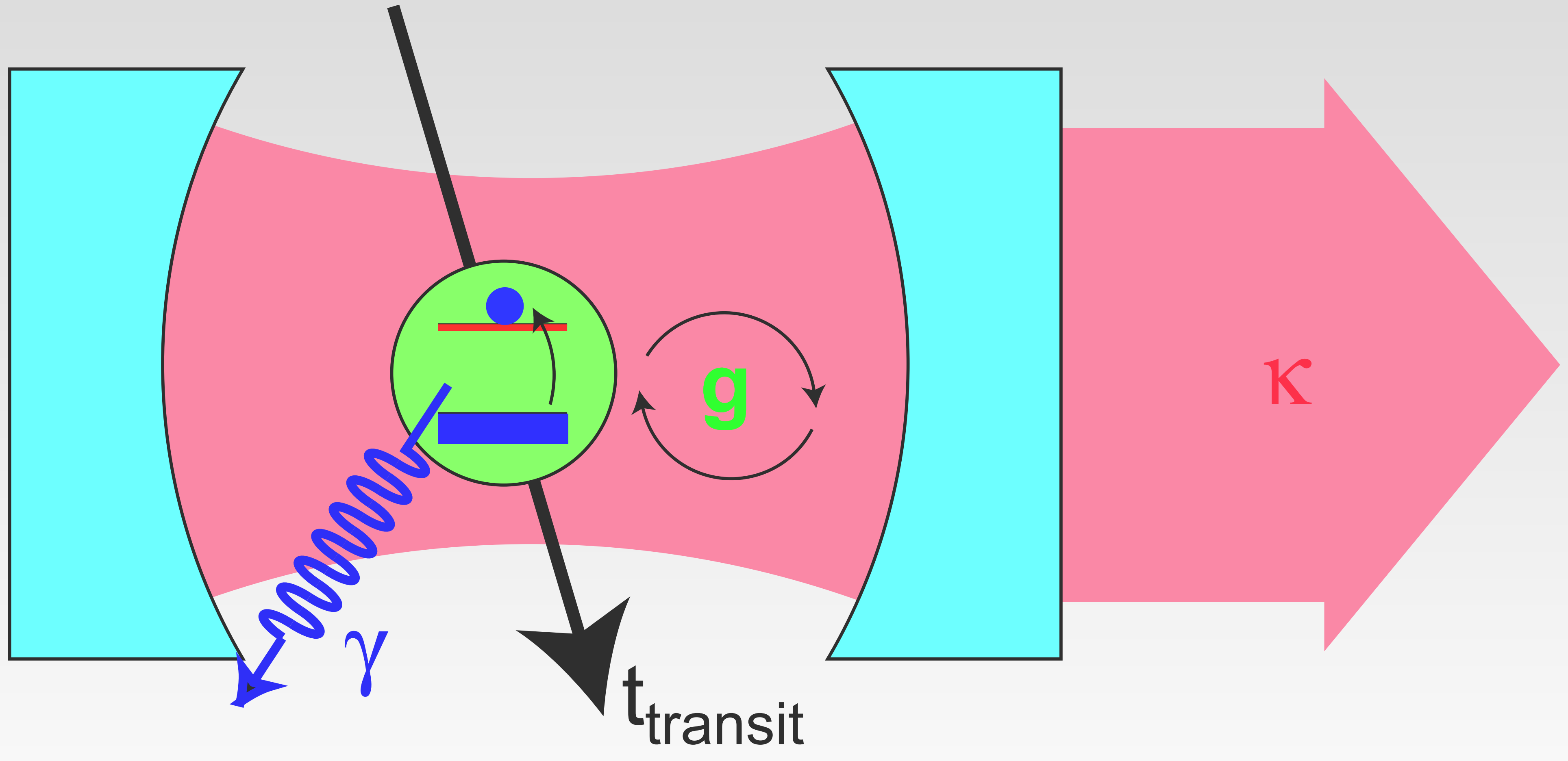


$$g \ll \omega$$

$$g(a^\dagger + a)(\sigma^+ + \sigma^-) \to \underbrace{g_1(a^\dagger\sigma^- + a\sigma^+)}_{\text{rotating}} + \underbrace{g_2(a^\dagger\sigma^+ + a\sigma^-)}_{\text{counter-rotating}}$$

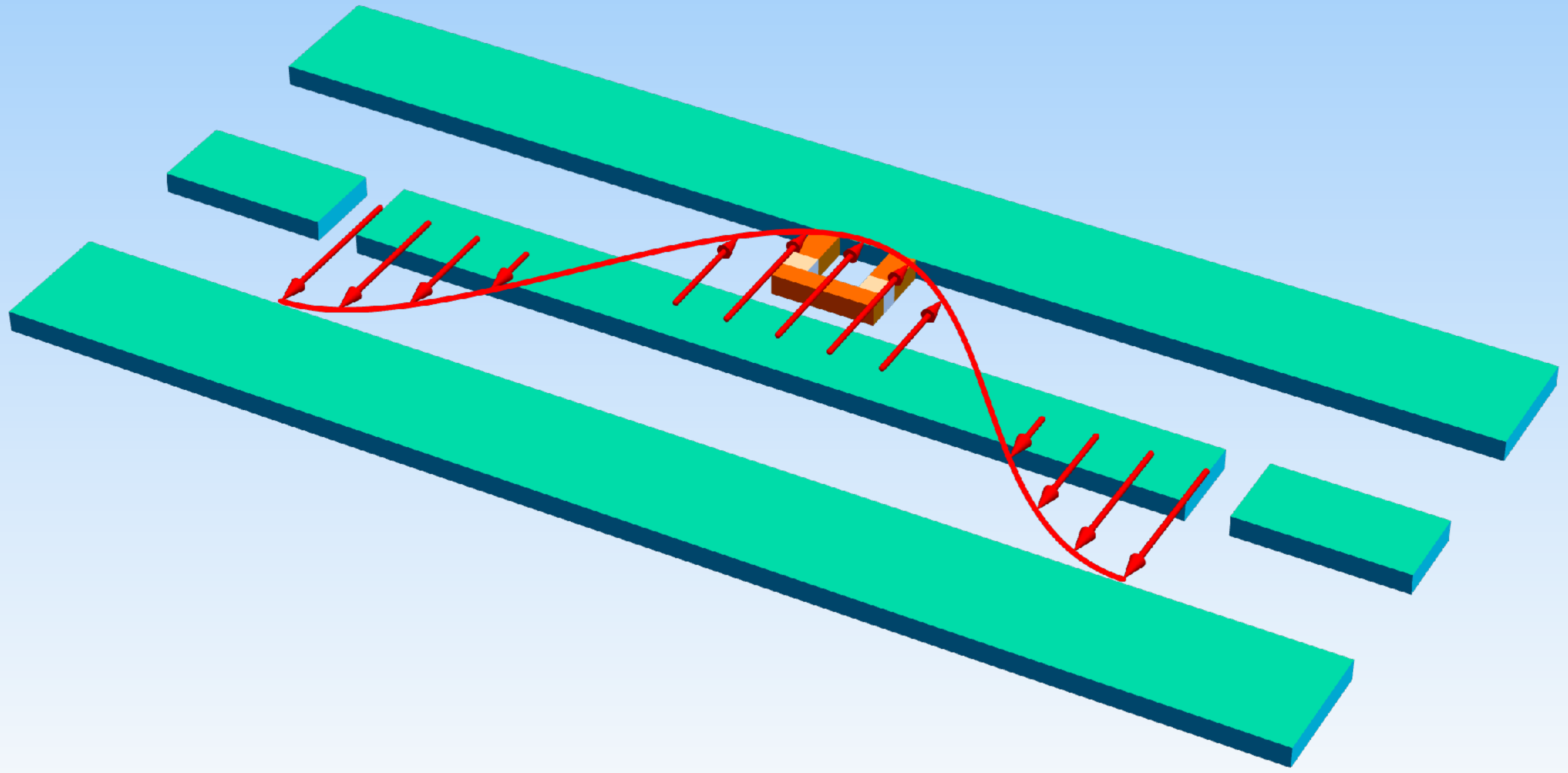
$$\kappa, \gamma \ll g \ll \omega$$

$$g = g_1 = g_2$$



$$g \sim \sqrt{\omega\Omega}$$

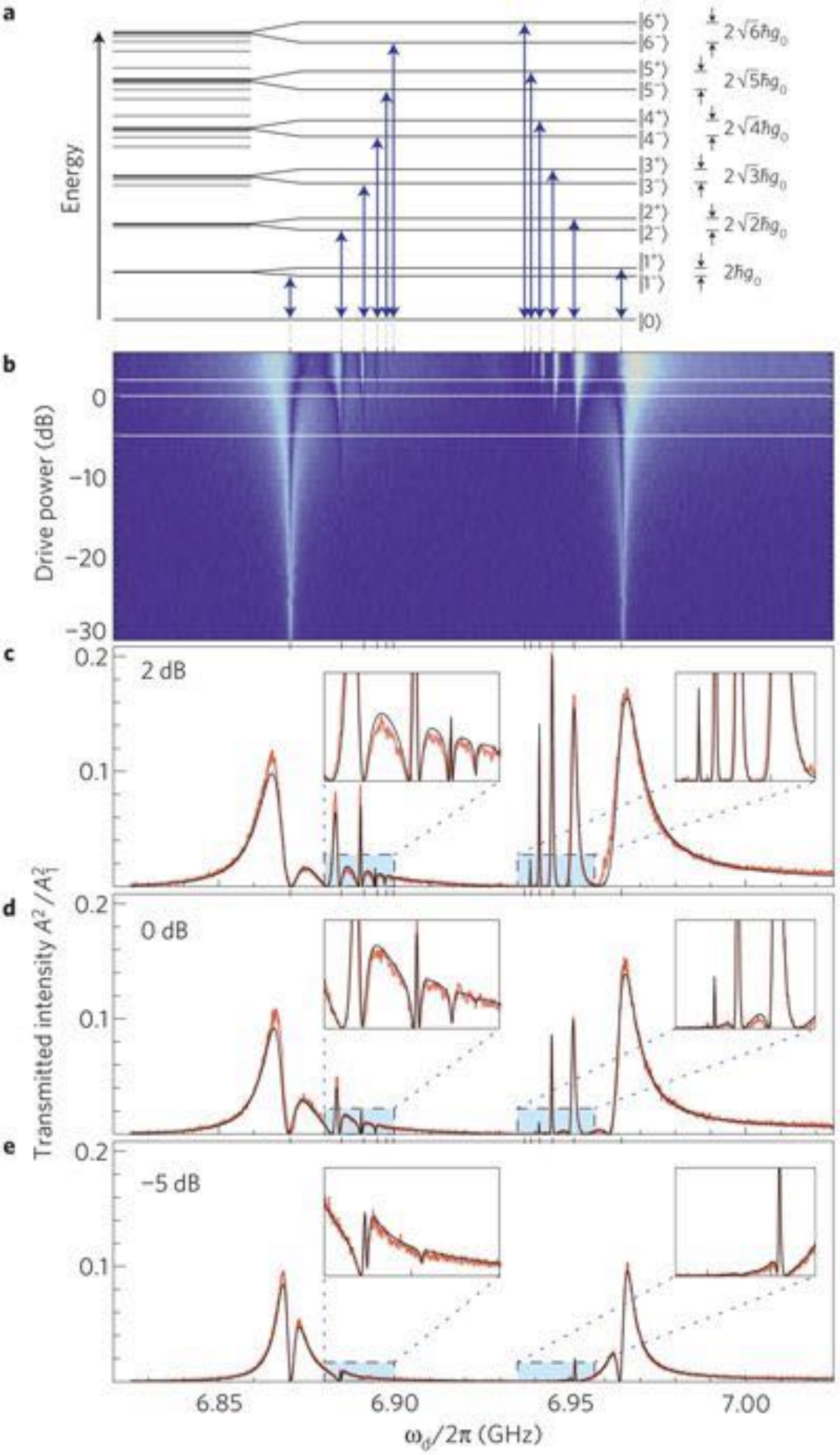
$$g_1 \neq g_2$$



ULTRA-STRONG COUPLING

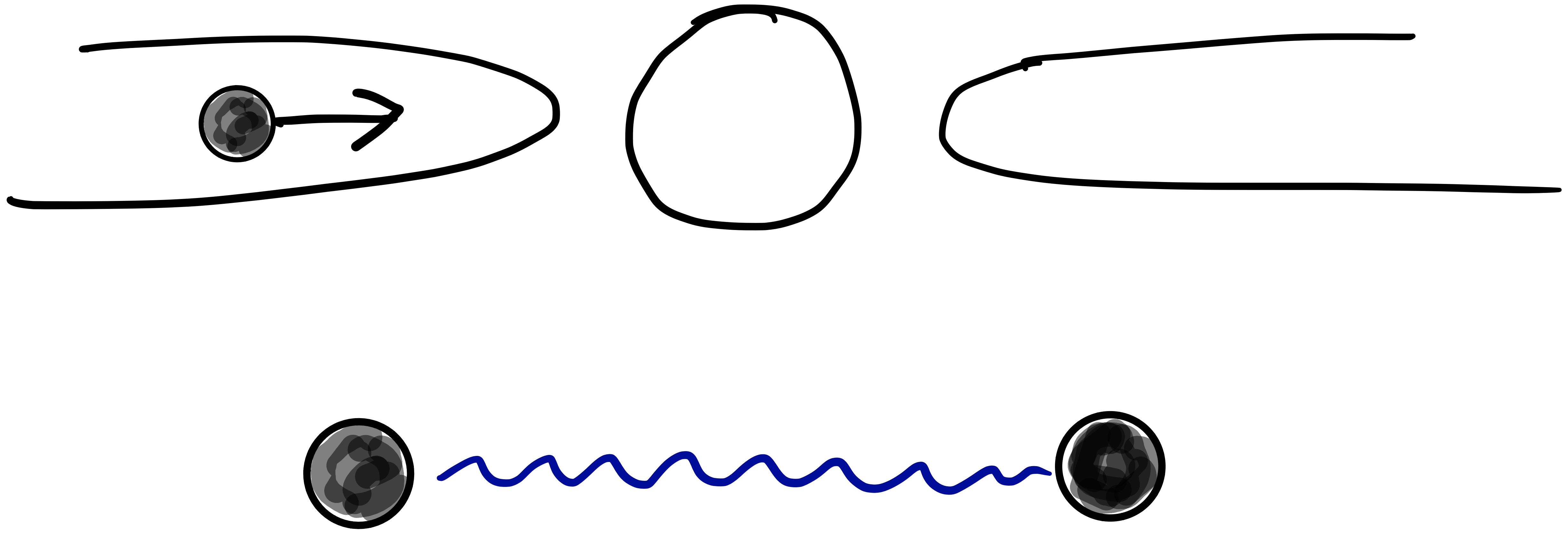
ADV. QUANT. TECH. 3, 2000085 (2020)

Bishop et al., Nature Physics (2009)
Bishop, PhD Thesis (2010)



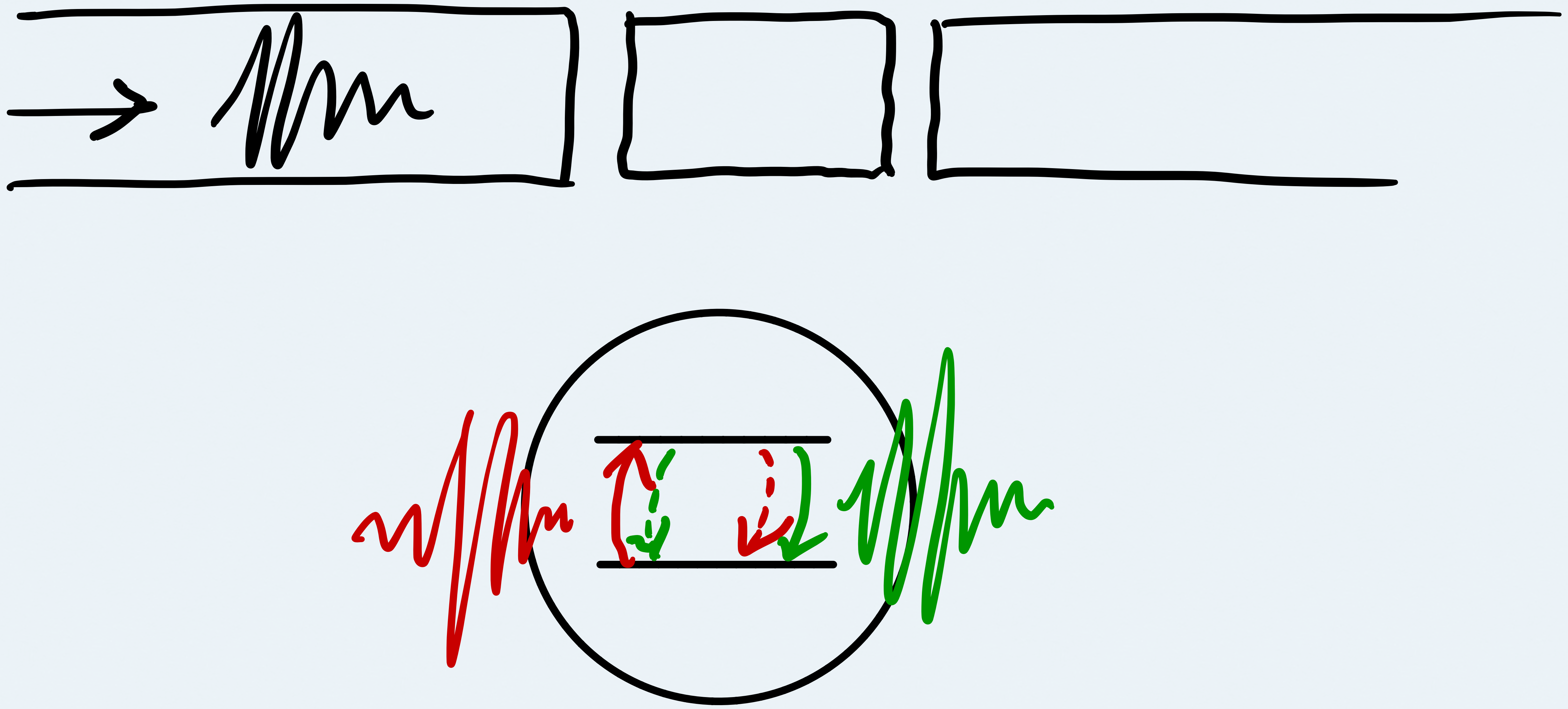
VACUUM RABI OSCILLATION

Bishop et al., Nature Physics (2009)
Bishop, PhD Thesis (2010)



$$\sim \frac{e^2}{2C} \equiv E_c$$

COULOMB BLOCKADE

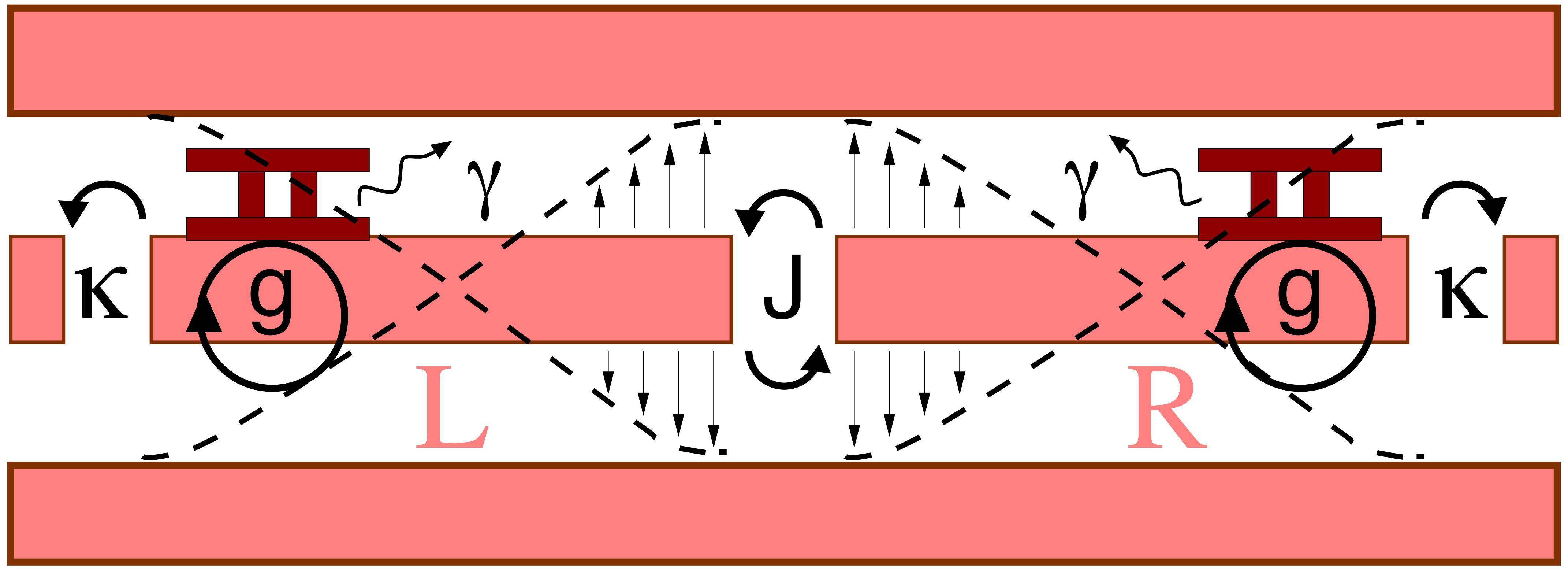


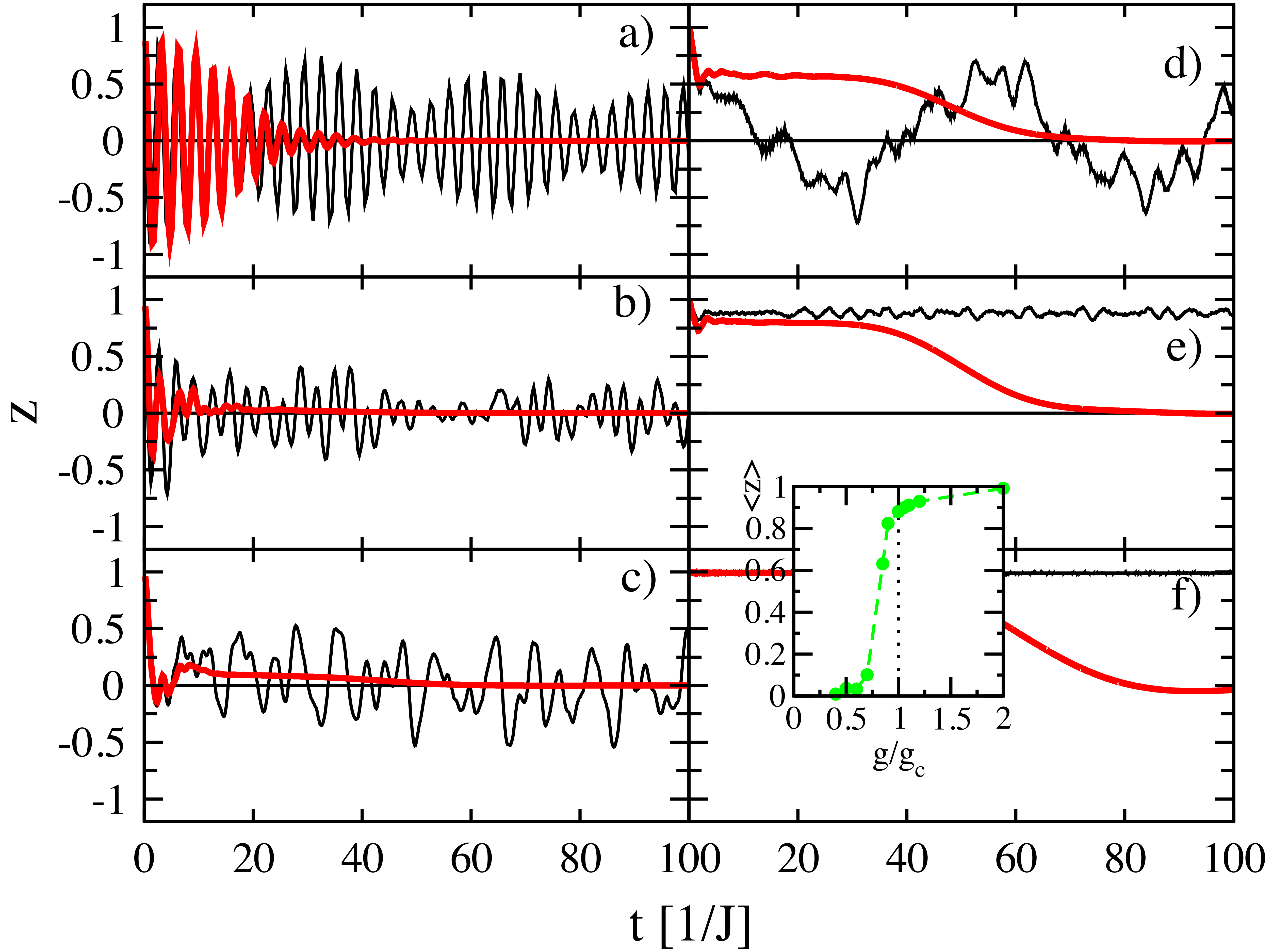
$$\sim g\,(a^\dagger + a)(\sigma^+ + \sigma^-)$$

PHOTON BLOCKADE

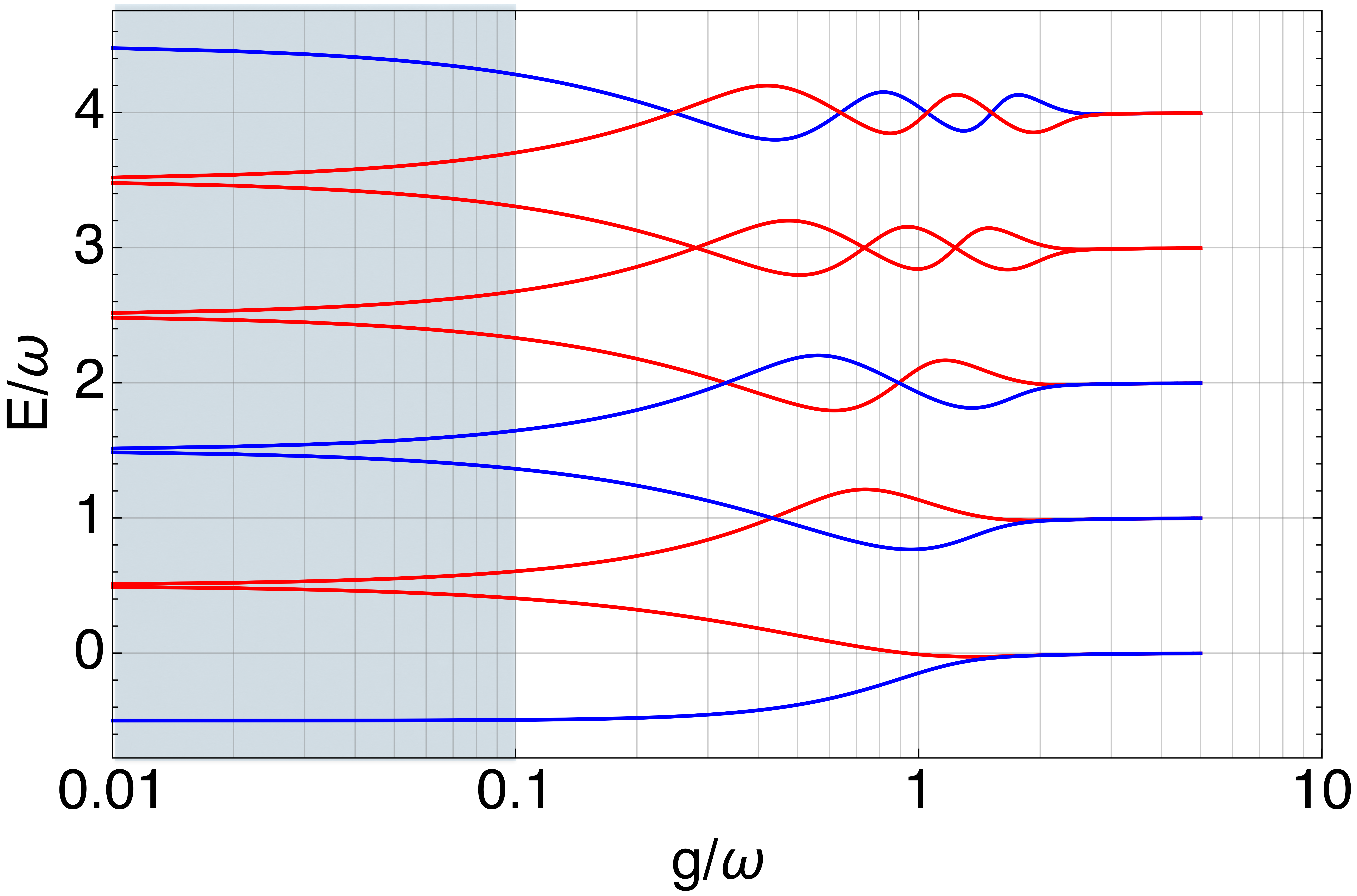
Photon Blockade

Schmidt *et al.*, (PRB, 2010); Raftery *et al.*, (PRX, 2014)

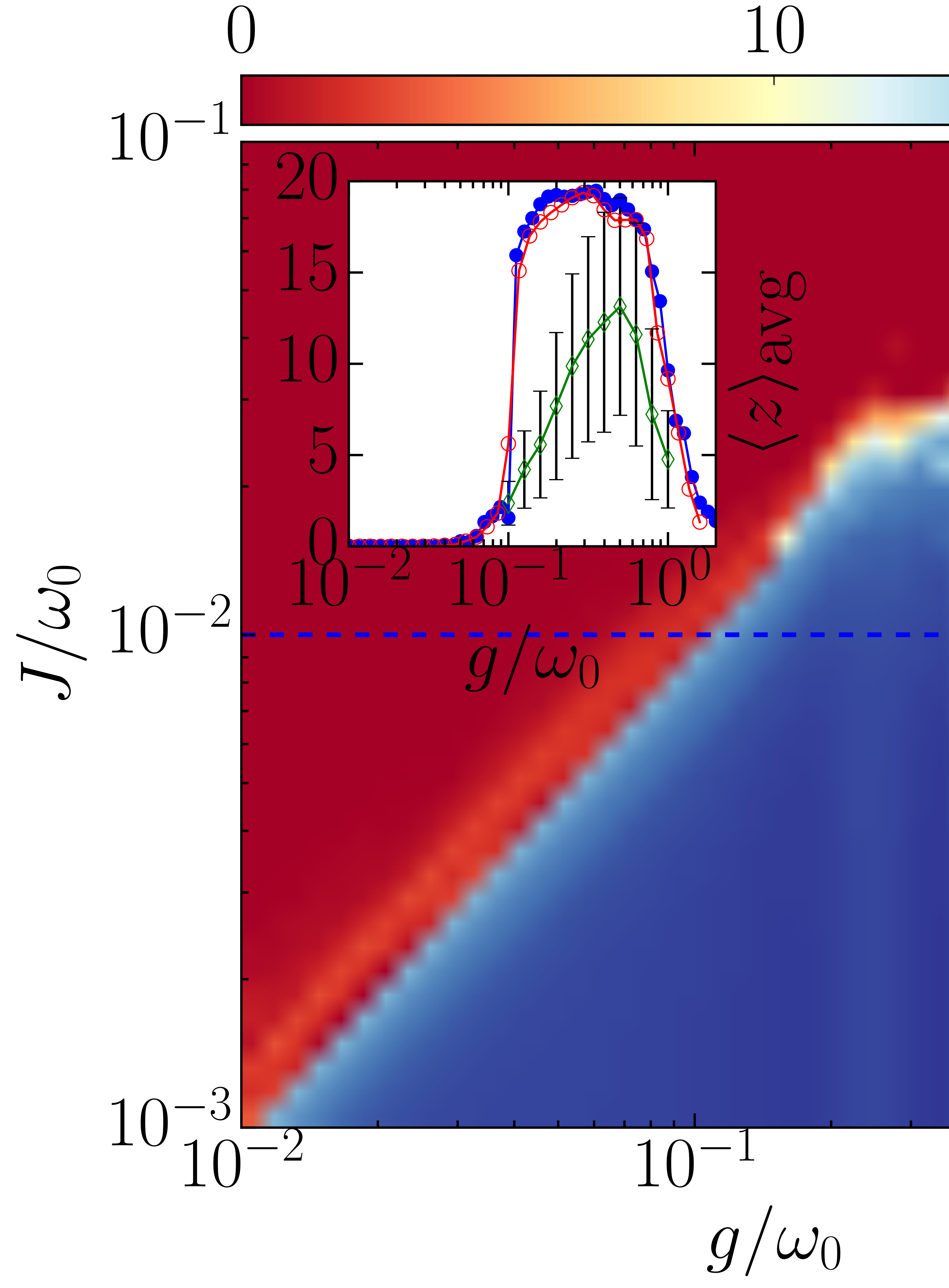




red curve: no decoherence, no photon loss
black curve: small decoherence+photon loss



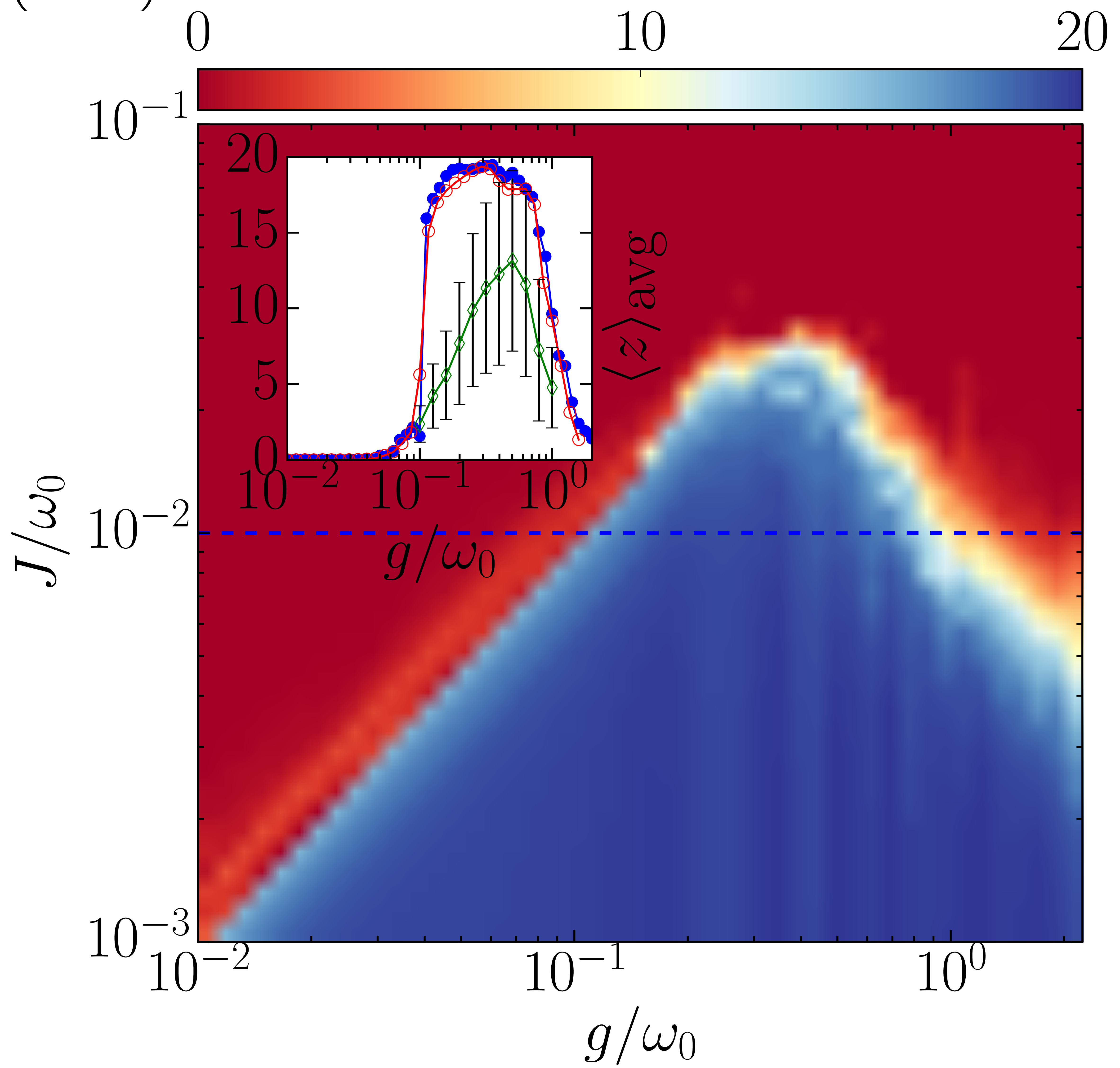
E/ω
4
3
2
1
0
0.01
0.1
1
10
g/ω

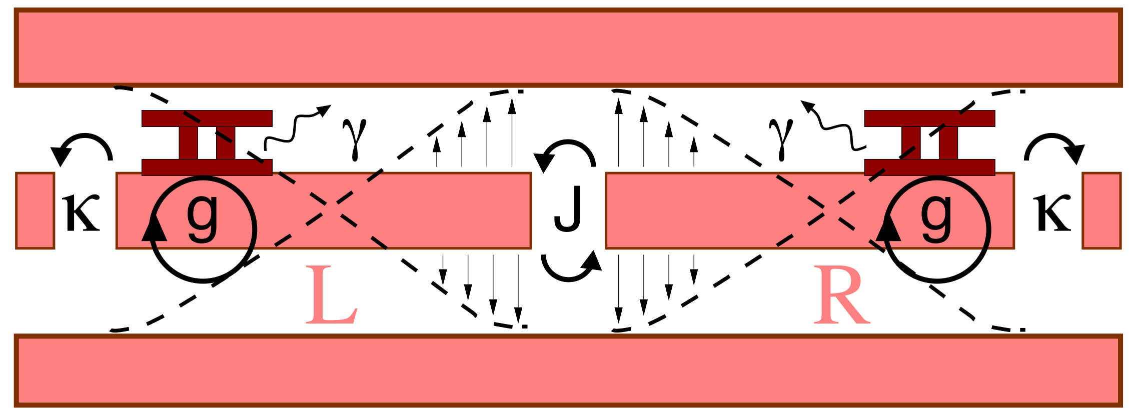


0
10
10^{-1}
10^{-2}
10^{-3}
J/ω_0
10^{-2}
10^{-1}
g/ω_0
20
15
10
5
0
$\langle z \rangle$avg
10^{-2}
10^{-1}
10^{0}
g/ω_0

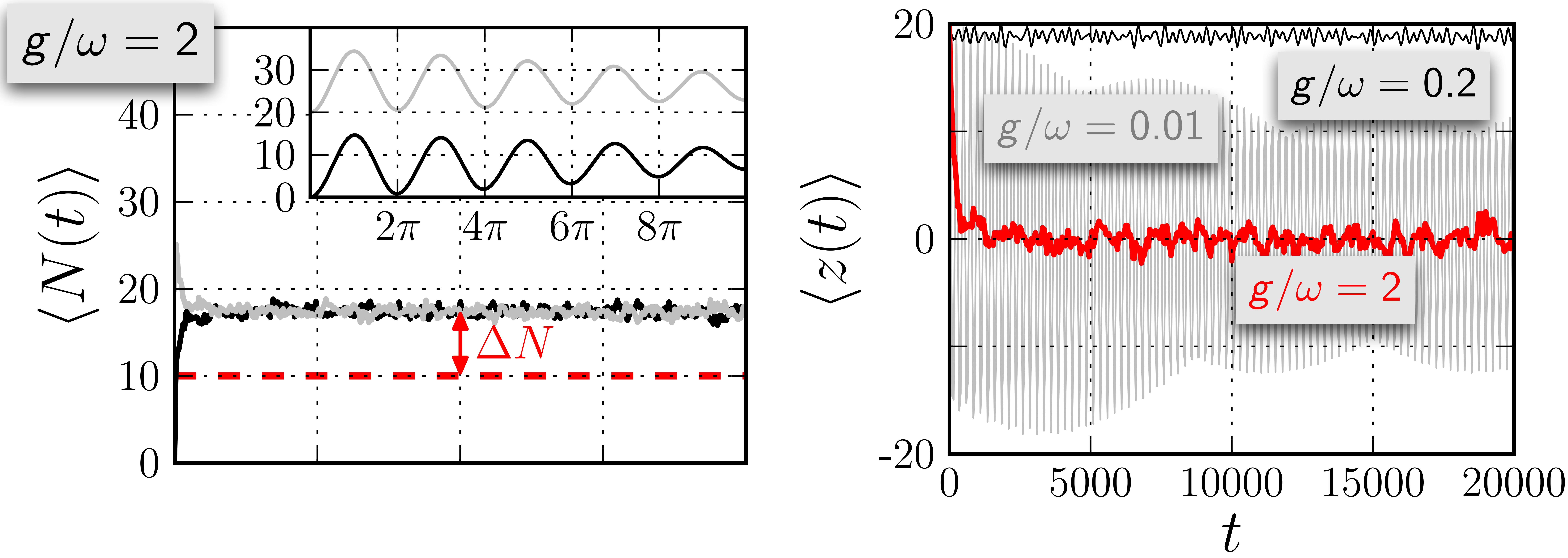
Recurrent Delocalization

Phys. Rev. Lett. 116, 153601 (2016)





QUASI-EQUILIBRIUM



Phys. Rev. Lett. 116, 153601 (2016)

Back to the Rabi Model

but from the (infinitely) strong-coupling limit

Infinitely Strong-Coupling Limit

$$\boxed{g \to \infty}$$

$$H_{\mathrm{Rabi}} = \omega a^\dagger a + \cancel{\frac{1}{2}\Omega\sigma^z} + g(a^\dagger + a)\sigma^x$$

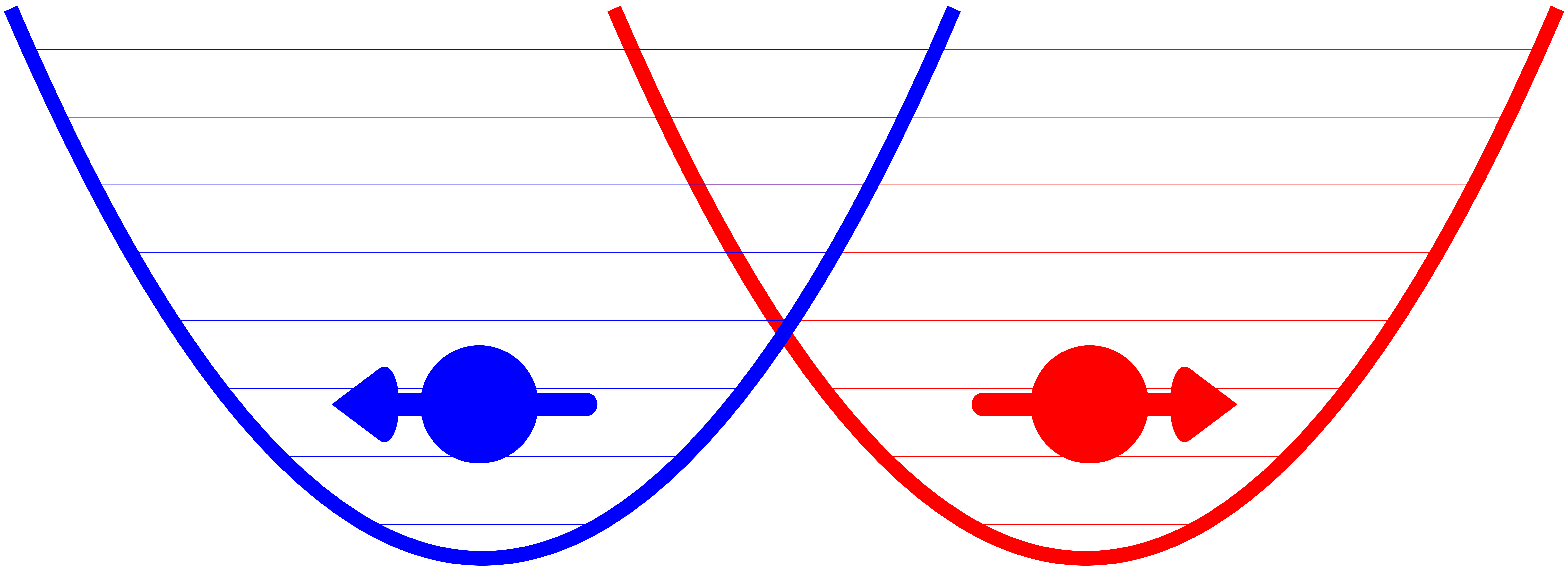
$$\boxed{|n, \pm\rangle = D(\pm g/\omega)\,|n\rangle \times |\pm\rangle}$$

$$D(z) \equiv \exp(za^\dagger - z^* a)$$

$$\boxed{E_{n,\pm} \approx \omega n + \mathcal{O}(e^{-g^2/2\omega^2})}$$

Infinitely Strong-Coupling Limit

$$g \to \infty$$



(Franck-Condon effect in polaron physics)

Parity Conserving Representation

Hwang & Choi (PRA, 2010; PRB, 2013)

$$H_{\mathrm{Rabi}} = \omega a^\dagger a + \frac{1}{2}\Omega\sigma^z + g(a^\dagger + a)\sigma^x$$

$$\boxed{\tau^z \equiv \cos(\pi a^\dagger a)\sigma^z}$$

$$\boxed{b \equiv a\sigma^x}$$

$$\boxed{H_{\mathrm{Rabi}} = \omega b^\dagger b - g(b^\dagger + b) + \frac{1}{2}\Omega\cos(\pi b^\dagger b)\tau^z}$$

Dressed States

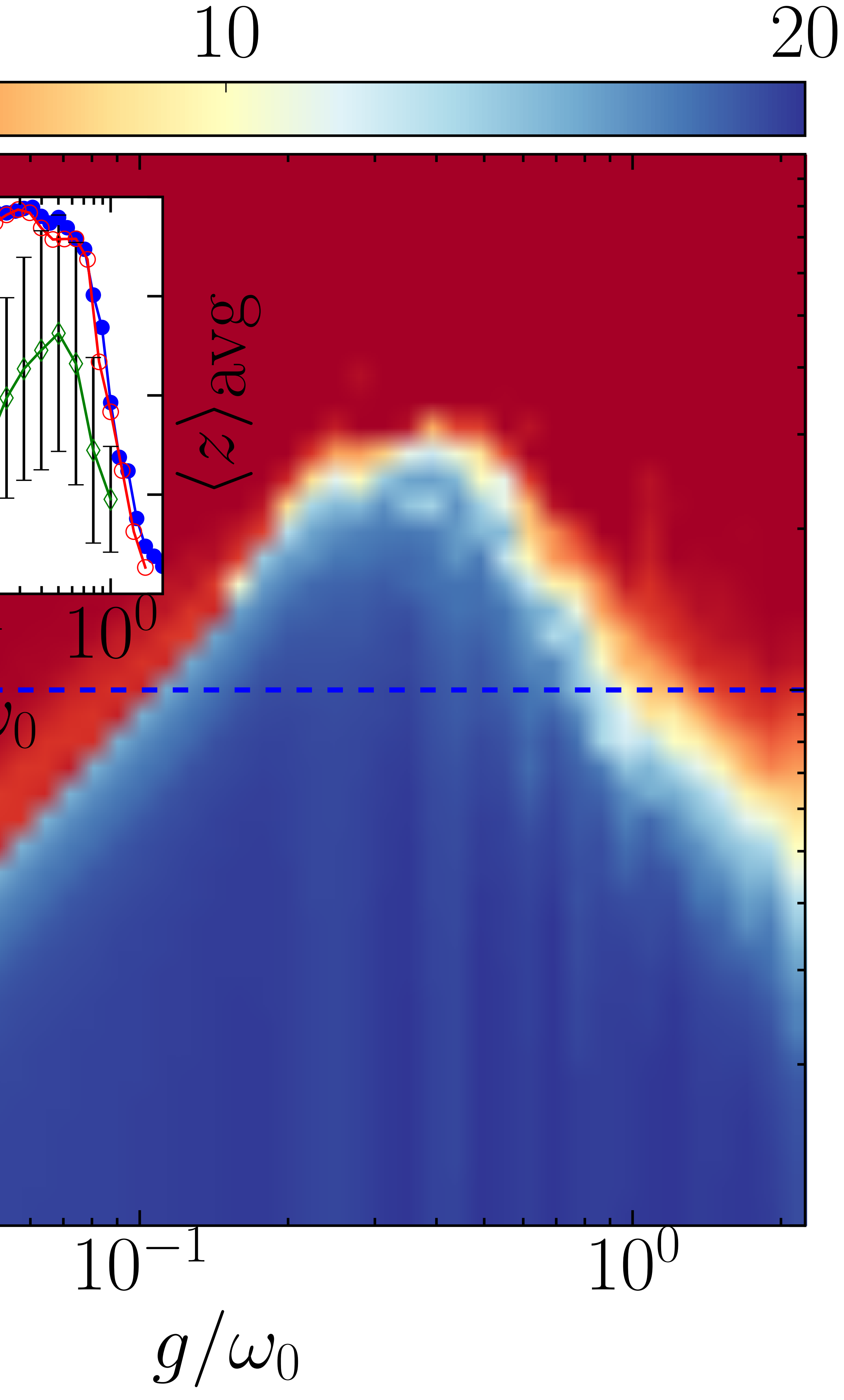
(Generalized Rotating Wave Approximation)

Feranchuk et al. (JPA, 1996); Irish (PRL, 2007)
Hwang & Choi (PRA, 2010; PRB, 2013)

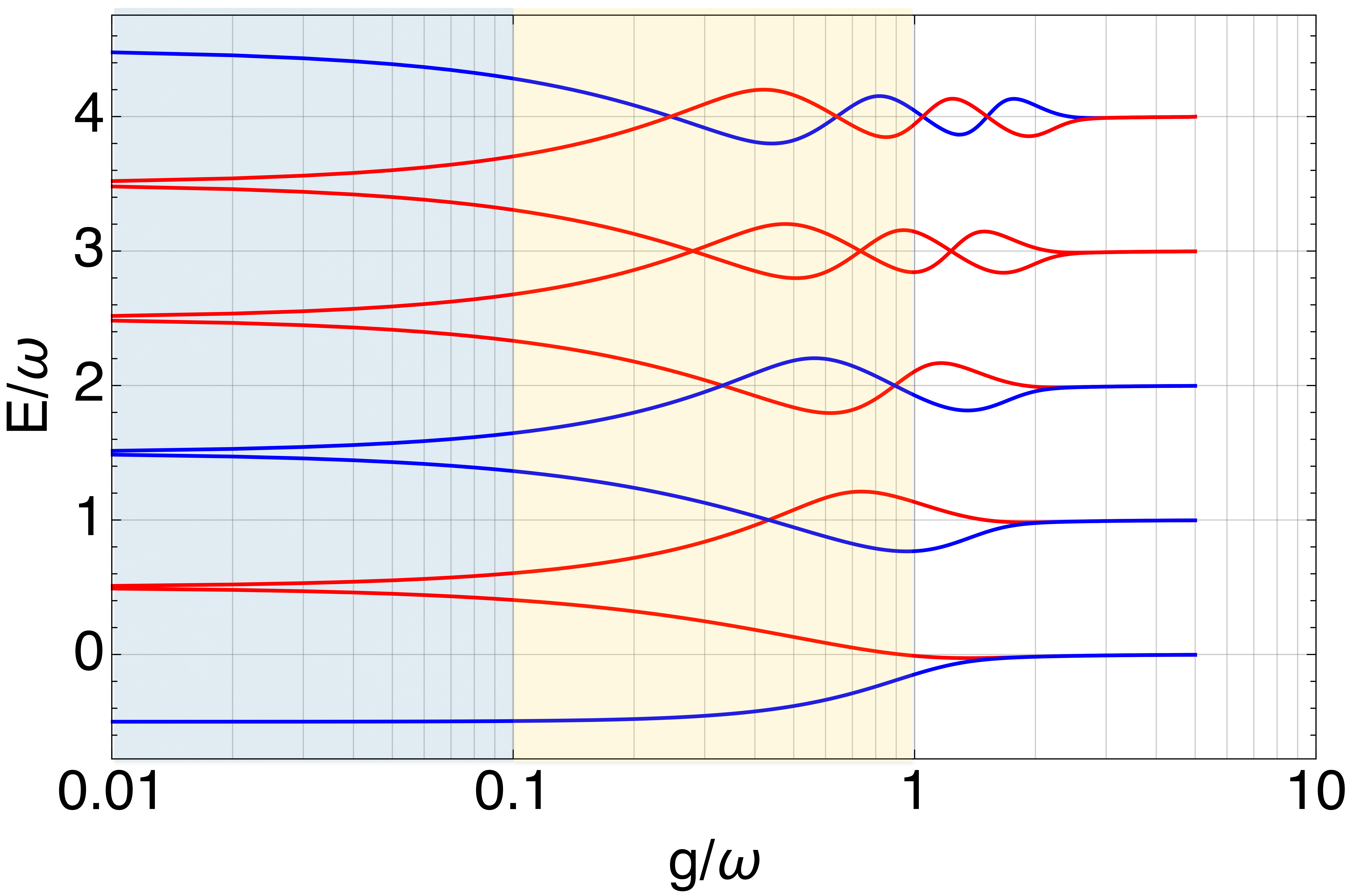
$$|n, e/o\rangle = D_b(g/\omega)\,|n\rangle_b \times |e/o\rangle$$

$$E_{n,e/o} \approx \omega n + \mathcal{O}(e^{-g^2/2\omega^2})$$

$$g \gg \omega$$



10
20
⟨z⟩avg
10^0
10^-1
10^0
g/ω0



E/ω
4
3
2
1
0
0.01
0.1
1
10
g/ω

Duality

Duality

Hwang & Choi (PRA, 2010)
Hwang, Puebla, & Plenio (PRL, 2015)

$$H \to \widetilde{H} = D^\dagger(\alpha) H D(\alpha)\,, \quad \alpha \equiv \frac{1}{2}\sqrt{\frac{\Omega}{\omega}\left(\frac{4g^2}{\omega\Omega} - \frac{\omega\Omega}{4g^2}\right)}$$

$$\widetilde{H} \approx \omega a^\dagger a + \frac{1}{2}\tilde{\Omega}\tilde{\sigma}^z - \tilde{g}(a^\dagger + a)\tilde{\sigma}^x$$

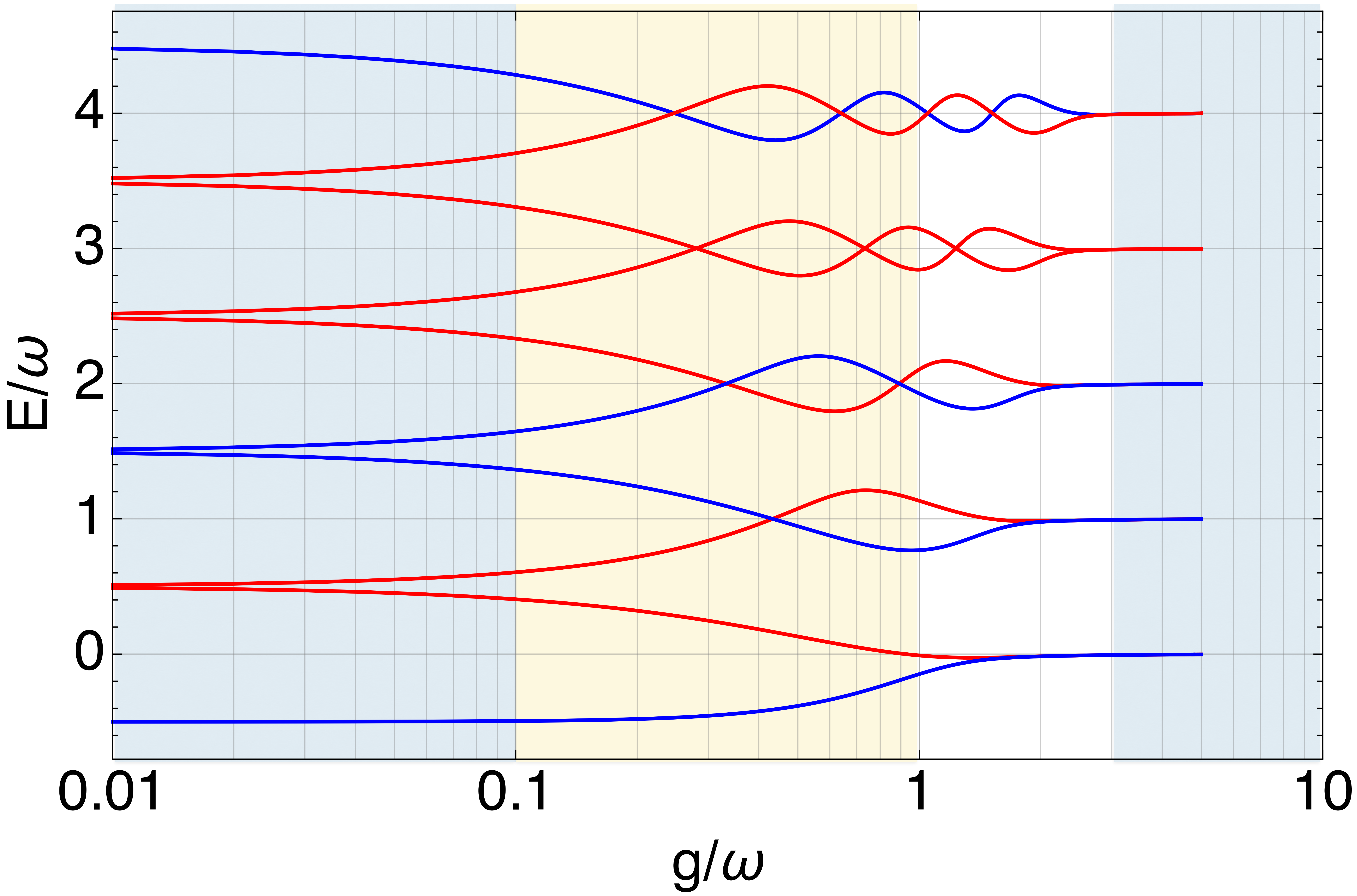
$$\boxed{\frac{\tilde{\Omega}}{\Omega} \equiv \frac{4g^2}{\omega\Omega}} \quad \boxed{\frac{\tilde{g}}{g} \equiv \frac{\omega\Omega}{4g^2}}$$

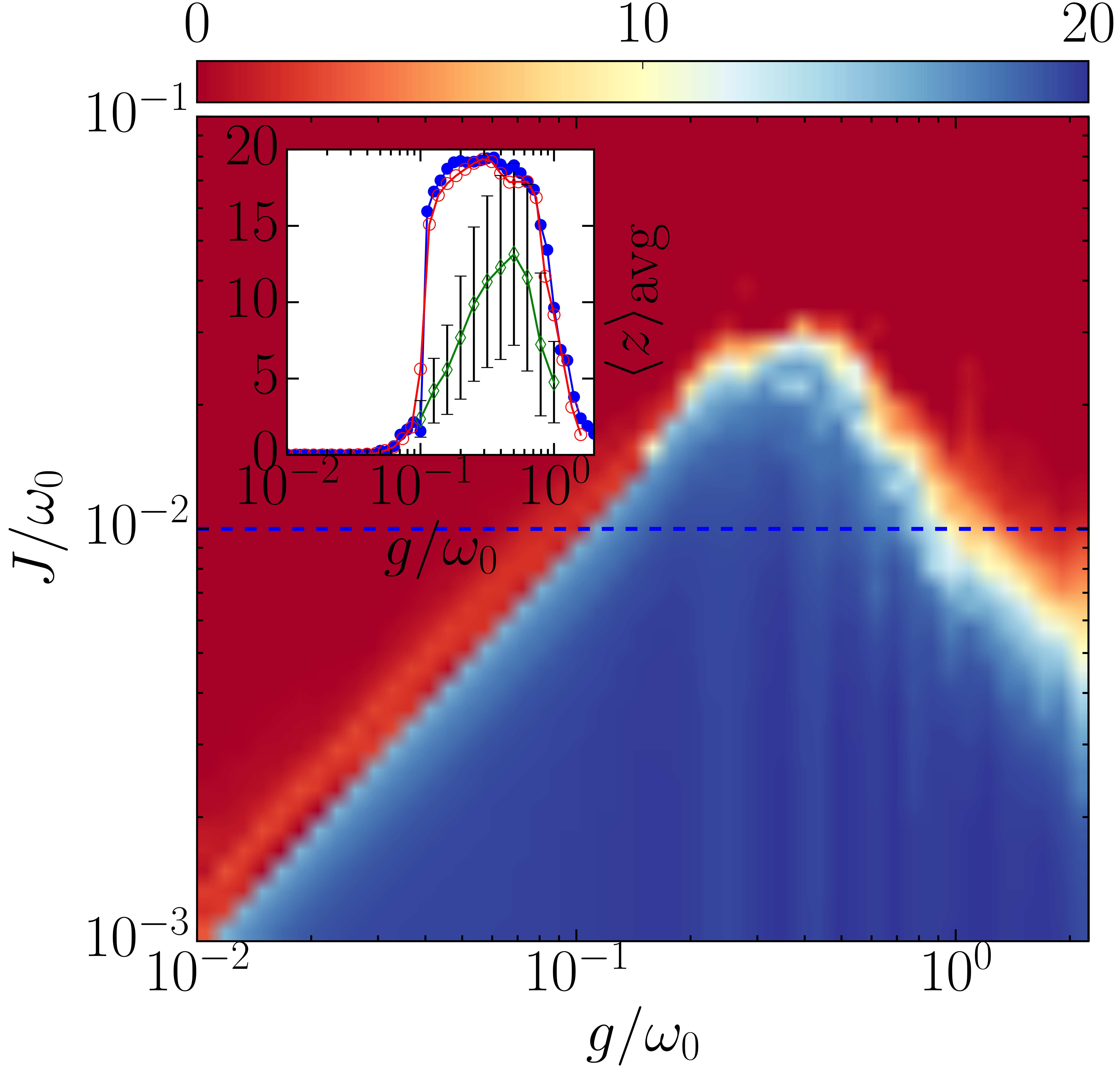
Duality!

Hwang & Choi (PRA, 2010)
Hwang, Puebla, & Plenio (PRL, 2015)

$$\widetilde{H} = \omega a^\dagger a + \frac{1}{2}\tilde{\Omega}\tilde{\sigma}^z - \tilde{g}(a^\dagger + a)\tilde{\sigma}^z$$

$$\boxed{\frac{2\tilde{g}^2}{\omega\tilde{\Omega}} = \frac{\omega\Omega}{2g^2}}$$


E/ω
4
3
2
1
0
0.01
0.1
1
10
g/ω



0
10
20
10^{-1}
10^{-2}
10^{-3}
J/ω_0
10^{-2}
10^{-1}
10^{0}
g/ω_0
20
15
10
5
0
$\langle z \rangle$avg

Thank you!