Summer school - Physics of Quantum Chips

June 30 – July 4, 2025 – University of Gdańsk, Gdańsk, Poland

Lectures, Peter Samuelsson, *Lund University*

1. **Quantum transport and tunneling in quantum dots.**
   - Landauer-Büttiker scattering approach to transport.
   - Double barrier structures and quantum dot tunneling.
   - Fermi's golden rule for tunnel rates.

Monday 30/6
9:00 – 10:30

2. **Density matrix description and quantum coherence.**
   - Density matrix – definition and properties.
   - Relaxation, dephasing and decoherence.
   - Quantum dot systems and charge coherence.

Tuesday 1/7
9:00 – 10:30

3. **Open quantum systems and Lindblad equation**
   - Lindblad equation, heuristic derivation.
   - Two-level system example.
   - Hybrid open quantum systems.

Wednesday 2/7
11:00 – 12:30

Dynamics of open quantum systems

Time evolution for closed system.

- System described by Hamiltonian H.
- For an individual state $|\psi_i\rangle$, with $U = e^{-iHt/\hbar}$ the unitary time evolution operator,

$$|\psi_i\rangle \rightarrow U|\psi_i\rangle \quad \Rightarrow$$

- For a density matrix

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \rightarrow \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger = U\rho U^\dagger$$

- Equation of motion (Liouville)

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H,\rho]$$

Time evolution for open system, described by POVM $\{E_k\}$, in terms of $L_k \sim E_k$

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\left[L_k^\dagger L_k \rho + \rho L_k^\dagger L_k\right]\right)$$

Derivation of Lindblad equation

Dephasing:
DQD dephasing in $|e\rangle, |g\rangle$ with pure dephasing time T_2^* .

Lindblad equation

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H,\rho] + L\rho L^\dagger - \frac{1}{2}\left(L^\dagger L\rho + \rho L^\dagger L\right) \qquad L = \sqrt{\frac{1}{T_2^*}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Longrightarrow \quad \rho(t) = \begin{pmatrix} \rho_{00} & \rho_{01}e^{-t/T_2^*} \\ \rho_{10}e^{-t/T_2^*} & \rho_{11} \end{pmatrix}$$

Relaxation: Lindblad operators
DQD relaxation in $|e\rangle, |g\rangle$ with relaxation time T_1 .

Lindblad equation

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H,\rho] + L\rho L^\dagger - \frac{1}{2}\left(L^\dagger L\rho + \rho L^\dagger L\right) \qquad L = \sqrt{\frac{1}{T_1}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

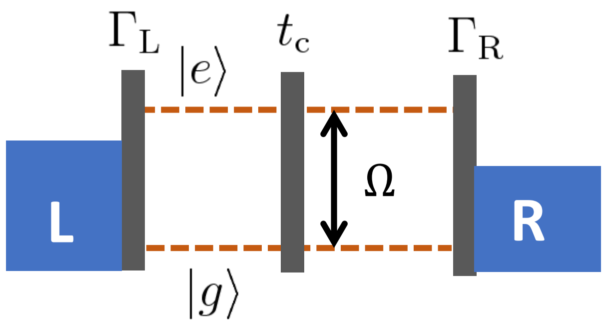
$$\Longrightarrow \quad \rho(t) = \begin{pmatrix} \rho_{00}e^{-t/T_1} & \rho_{01}e^{-t/T_2} \\ \rho_{10}e^{-t/T_2} & \rho_{11} + \rho_{00}\left(1 - e^{-t/T_1}\right) \end{pmatrix} \qquad T_2 = 2T_1$$

Calculation if time permits

Double quantum dot beyond qubit

DQD coupled to L,R leads (rotating frame)

- DQD Hilbert space: $|e\rangle, |g\rangle, |0\rangle$ (empty).



Lindblad equation for density matrix $\hat{\rho}$

$$\partial_t \hat{\rho} = \Gamma_{0e}\mathcal{D}[|0\rangle\langle e|]\hat{\rho} + \Gamma_{g0}\mathcal{D}[|g\rangle\langle 0|]\hat{\rho} + \Gamma_{\varphi}\mathcal{D}[\hat{\sigma}_z]\hat{\rho} + \gamma\mathcal{D}[\hat{\sigma}_-]\hat{\rho}$$

tunneling out of $|e\rangle$

tunneling in to $|g\rangle$

double dot dephasing

double dot relaxation

Dissipator

$$\mathcal{D}(A)\rho = A\rho A^\dagger - \frac{1}{2}\left(A^\dagger A\rho + \rho A^\dagger A\right)$$

Similar style for DQD coupled to L,R leads and microwave resonator