Summer school - Physics of Quantum Chips

June 30 – July 4, 2025 – University of Gdańsk, Gdańsk, Poland

Lectures, Peter Samuelsson*, Lund University*

1. **Quantum transport and tunneling in quantum dots.**
   - Landauer-Büttiker scattering approach to transport.
   - Double barrier structures and quantum dot tunneling.
   - Fermi's golden rule for tunnel rates.

Monday 30/6
9:00 – 10:30

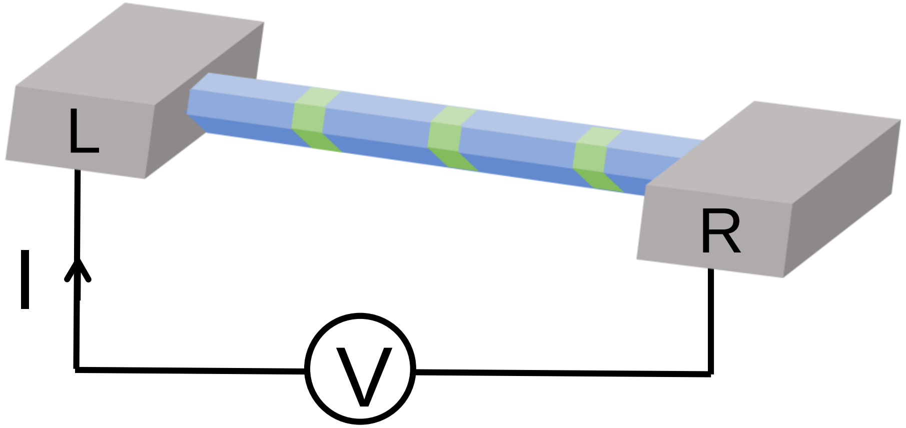
2. **Density matrix description and quantum coherence.**
   - Density matrix – definition and properties.
   - Relaxation, dephasing and decoherence.
   - Quantum dot systems and charge coherence.

Tuesday 1/7
9:00 – 10:30

3. **Open quantum systems and Lindblad equation**
   - Lindblad equation, heuristic derivation.
   - Two-level system example.
   - Hybrid open quantum systems.

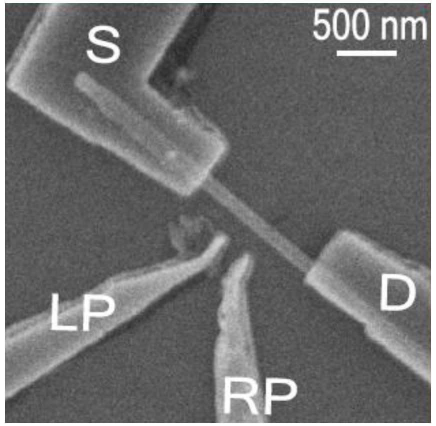
Wednesday 2/7
11:00 – 12:30

Quantum transport - Landauer

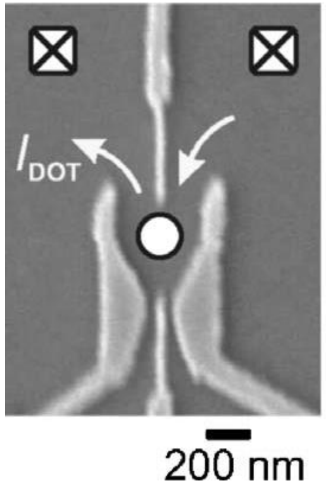


- Quantum conductor coupled to two electronic terminals, L and R.
- Terminals in thermodynamic equilibrium, at temperatures θ_L, θ_R and chemical potentials μ_L, μ_R.
- Applied voltage bias $eV = \mu_L - \mu_R$.
- Electrical current I.

QUANTUM DOT EXAMPLES



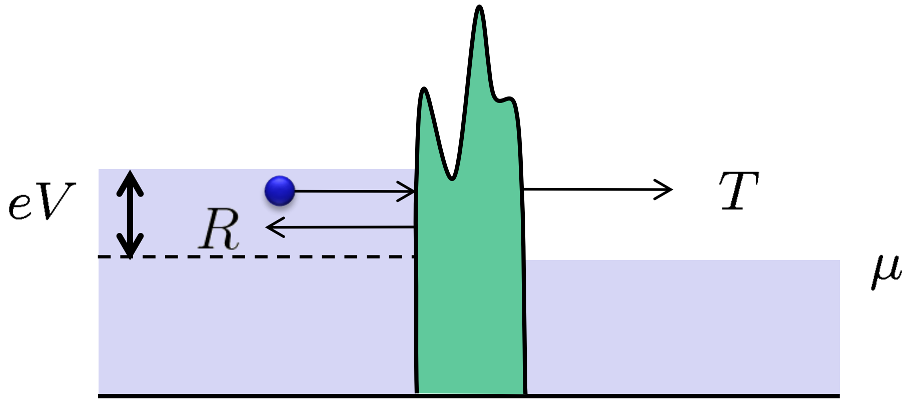
Nanowire double quantum dot conductor.
Khan et al, Nat. Commun. 2021.



Single quantum dot conductor in two-dimensional electron gas.
Hanson et al, Rev. Mod. Phys. 2007.

Conductance from transmission

Heuristic 1D discussion, scattering at potential barrier, zero temp. $\theta_L, \theta_R = 0$.



- Left terminal, $\mu_L = \mu + eV$.
- Right terminal, $\mu_R = \mu$.
- Transmission probability, T.
- Reflection probability, $R = 1 - T$.

$$\text{charge} \times \frac{\text{length}}{\text{time}} \times \frac{1}{\text{length}} = \frac{\text{charge}}{\text{time}}$$

$I_{in} = ev_F n$ incident current from L

$I = I_{in} T$ net current (transmitted part)

v_F Fermi velocity $v_F = p_F/m = \hbar k_F/m$

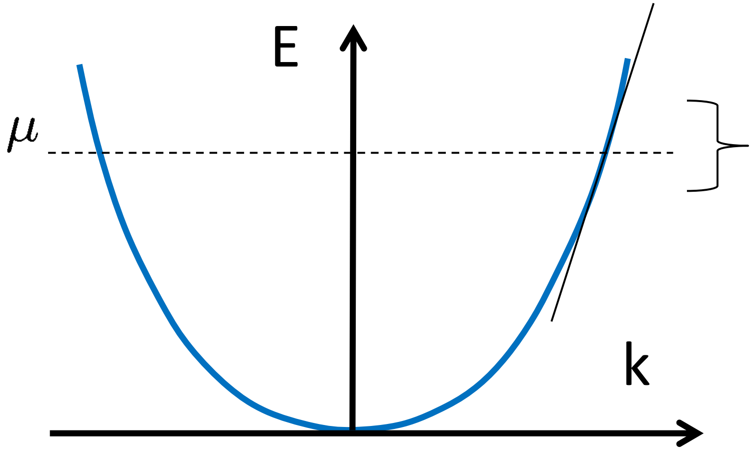
$n = \frac{d\rho}{dE} eV$ electron density (number of electrons per unit length)

density of states (per unit length and energy)

energy interval

$$E = \frac{\hbar^2 k^2}{2m}$$

Dispersion of free electrons



Relevant energy scales $k_B\theta, eV$ small compared to chemical potential μ.

$$E \approx \mu + \hbar v_F (k - k_F)$$

Linearized dispersion around Fermi energy $E_F = \mu$

$$\frac{d\rho}{dE} = \frac{\partial\rho}{\partial k}\frac{\partial k}{\partial E} = \frac{1}{2\pi}\frac{1}{\hbar v_F}$$

1D - density of states

Rightmoving electrons: half of «particles in a box»

$$kL = n\pi \Rightarrow \frac{\partial\rho}{\partial k} = \frac{1}{2\pi}$$

$$\Rightarrow I_{in} = e v_F \frac{\partial\rho}{\partial E} eV = \frac{e^2}{h} V$$

independent of system properties

$$G_0 = \frac{e^2}{h}$$

Current and conductance

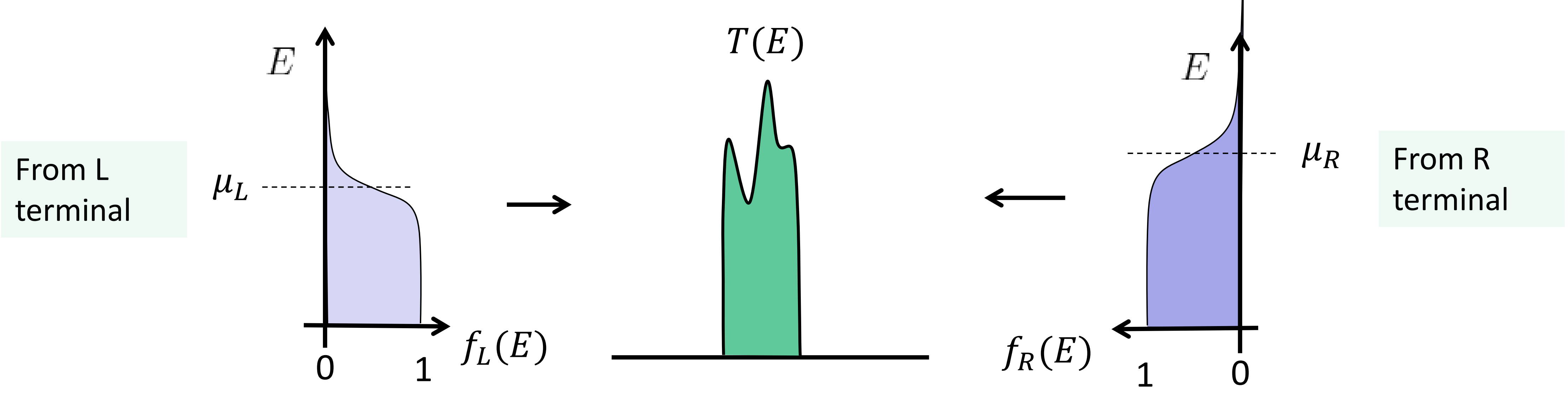
$$I = (e/h)TeV \Rightarrow G = dI/dV = \frac{e^2}{h} T$$

Landauer formula

Conductance quanta (single spin)

Extending to finite terminal temperatures

Terminals serve as sources of electron streams



- Occupations determined by terminal Fermi-Dirac distributions

$$f_{L/R}(E) = \frac{1}{1 + e^{(E-\mu_{L/R})/k_B\theta_{L/R}}}$$

- Electrical current flow from L to R and R to L

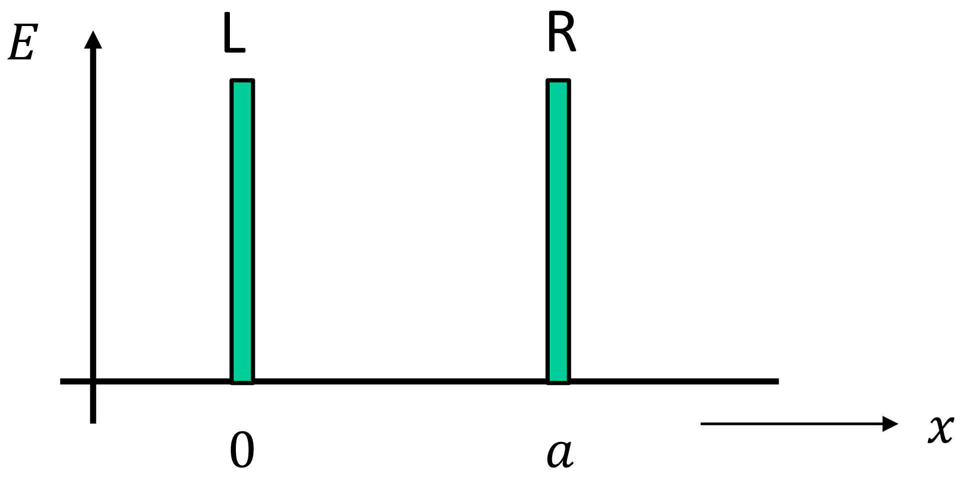
$$I_{L\to R} = \frac{e}{h}\int dE\ T(E) f_L(E), \qquad I_{R\to L} = \frac{e}{h}\int dE\ T(E) f_R(E)$$

- Net current $I = I_{L\to R} - I_{R\to L} = \frac{e}{h}\int dE\ T(E)[f_L(E) - f_R(E)]$

Landauer formula recovered at small voltage and temp.

Quantum dot transmission

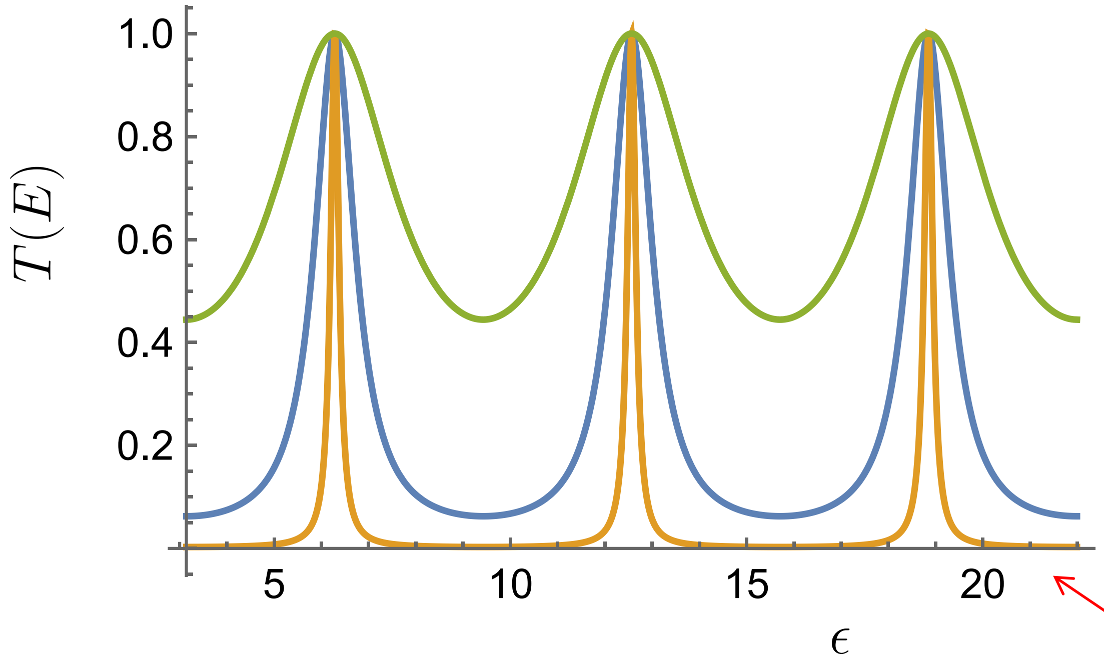
Transmission probability, 1D double barrier structure as quantum dot



- Scattering amplitudes (energy indep., symmetric): t_L, r_L, t_R, r_R.
- Linearized dispersion around μ as $E = \mu + \hbar v_F(k - k_F)$.
- Probabilities and conservation $T_{L/R} = |t_{L/R}|^2, R_{L/R} = |R_{L/R}|^2$, $T_{L/R} + R_{L/R} = 1$.
- Tunneling limit $T_{L/R} \ll 1$.

- Calculate total transmission probability,
- Identify tunnel rates.

Example with $T_L = T_R = 0.8, 0.4, 0.1.$



$$\epsilon = \frac{2a(E-\mu)}{\hbar v_F} + \phi$$

Transmission maxima at

$\epsilon = 2n\pi$ ⇒

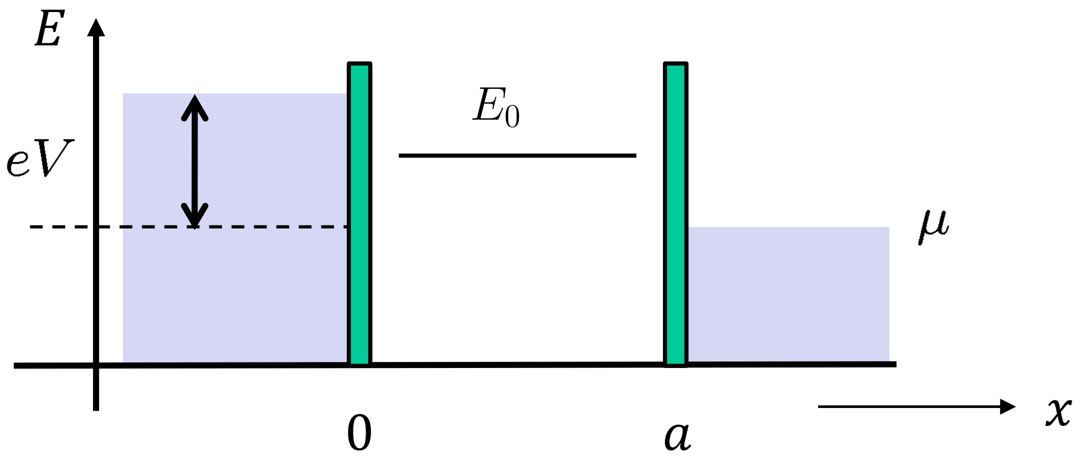
$$E = \mu + \frac{\hbar v_F}{a}(n\pi - \phi/2)$$

Narrow resonances in tunnel limit

Compare to particle-in-a-box

$k = \frac{n\pi}{a}$ ⇒ $E = \mu + \frac{\hbar v_F}{a} n\pi$

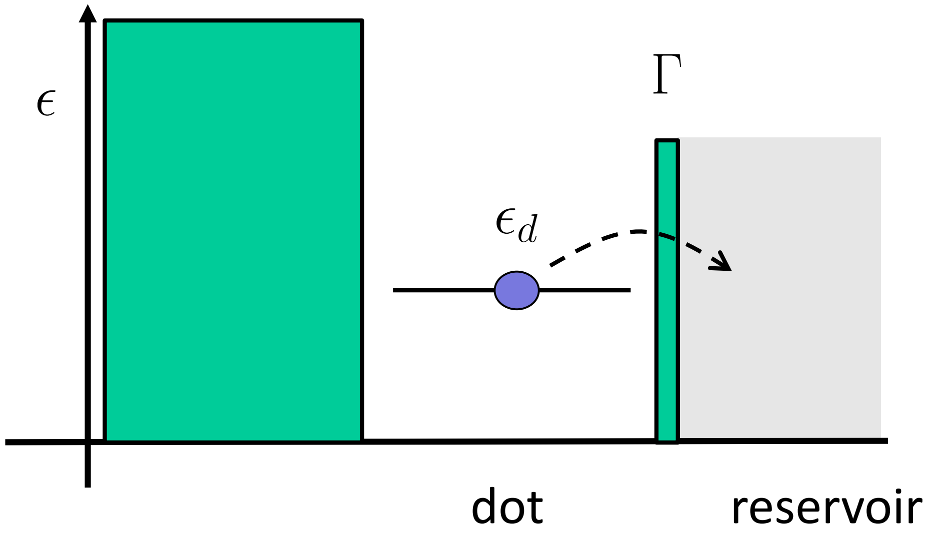
Current for large voltage bias and zero temperature



$$I = \frac{e}{h} \int dE T(E) \left[f_L(E) - f_R(E) \right] = \ldots = e \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

Fermi's golden rule for tunnel rate

Tunnel-out rate for an electron in a quantum dot level, coupled to a reservoir.



Wavefunction

$$|\Psi(t)\rangle = a_d(t)|d\rangle + \int d\epsilon a_\epsilon(t)|\epsilon\rangle$$

Normalization

$$|a_d(t)|^2 + \int d\epsilon |a_\epsilon(t)|^2 = 1$$

Initial conditions

$$a_d(0) = 1, a_\epsilon(0) = 0$$

Hamiltonian description

$$H = \underbrace{\epsilon_d |d\rangle\langle d| + \int d\epsilon \epsilon |\epsilon\rangle\langle\epsilon|}_{H_0} + \underbrace{\int d\epsilon \left[t_\epsilon |d\rangle\langle\epsilon| + t_\epsilon^* |\epsilon\rangle\langle d|\right]}_{H'}$$

Calculate tunnel rate